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A Note on Effective Teaching and Interpretation of Compound Return Measures of Investment Performance

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Abstract

This teaching note illustrates and highlights two measurements of investment performance, geometric mean and internal rate of return. Examples are used to show that the internal rate of return is just a special case of the geometric mean. Additionally, suggestions are made for renaming these measures to clarify for students and investors their similarities and appropriate applications of each.

I. Introduction

Many students of investment management and individual investors find the accurate computation and interpretation of investment returns a difficult task. Textbooks and industry performance norms have attempted to address these issues by providing explicit instruction on the appropriate methods for measuring investment performance. Two of the primary return measures are the geometric mean return and the internal rate of return. While these are fundamental methods of evaluating investment performance, students and investors often exhibit little understanding of how to correctly interpret the numerical solutions resulting from their computation.

This note is designed to illustrate and highlight these two measures of investment performance. Part of the problem in understanding the similarities and differences in geometric mean and internal rate of return seems to stem from the labels the profession has attached to each. Examples are used to show that, in fact, the internal rate of return is just a special case of the geometric mean, and suggestions are made for renaming these measures to clarify their similarities and appropriate applications of each.

II. The Arithmetic Mean Return

Students often find the calculation of investment returns initially challenging. First, they must grasp holding period return (HPR) as a measure for a single investment period [1] They must next utilize HPRs to compute measures of performance over multiple periods.

When asked to compute a measure of average return for multiple periods, students often initially calculate a simple average of the periodic HPRs. The arithmetic mean return is given by (1).

[1] See the footnote in Table 1 for the calculation of a single Holding Period Return.

$$R_A = \Sigma (R_t) / n \qquad (1)$$

Here, R_t is the periodic holding period return for each of n periods. While useful as a measure of average performance across different securities (such as an industry average) and as an unbiased forecast of next period expected return, the arithmetic mean ignores compounding. This makes arithmetic mean return a poor measure as a historical, comparative performance statistic.

III. The Geometric Mean Return

Kosmicke and Opsal (1988) state that the *geometric mean return* is the correct measure of fundamental equivalence among stocks. The geometric mean is the compound average annual return which results in the same cumulative dollars as compounding by each individual holding period return.[2] The geometric mean return is given by (2).

$$R_G = [\Pi (1 + R_t)]^{1/n} - 1 \qquad (2)$$

Table I illustrates the calculation of the geometric mean return on investment.

Table I: Illustration of Calculation of Geometric Mean Return

Year	Ending Net Asset Value	Income	Holding Period Return*
1996	\$18.25	----	-----
1997	\$22.75	0.35	26.58%
1998	\$21.10	0.35	–5.71%
1999	\$26.35	0.50	27.25%
2000	\$29.90	0.50	15.37%

$R_G = [(1.2658)(.9429)(1.2725)(1.1537)]^{1/4} - 1 = 15.05\%$

**Holding Period Return* is computed as

[Ending NAV - Beginning NAV + Income]/Beginning NAV,

where NAV NAV is net Asset Value per Share, and Income includes dividends, interest and capital gains distributions.

The hypothetical mutual fund in Table I had a compound average annual return of 15.05 percent over the four-year period 1997-2000. Had an investor invested \$10,000 in the fund at the beginning of 1997 and reinvested all income, by the end of 2000 the fund investment would have

[2] Accurate performance measurement should be based on the appropriate period, as in daily, monthly, or quarterly returns. For simplicity of demonstration, all periodic returns here are treated as annual returns.

been worth \$17,521.91. The following shows the equivalency of the geometric mean return and the cumulative periodic compounding of the holding period returns.

$$\$10{,}000(1.2658)(.9429)(1.2725)(1.1537) = \$17{,}521.91$$
$$\$10{,}000(1.1505)^4 = \$17{,}520.50^3$$

Investment management textbooks refer to the geometric mean return as a time-weighted measure of performance.[3] Implicit in the calculations shown above is the fact that a lump sum was initially invested (here, \$10,000) and left intact during the life of the investment. No additional dollars were invested, and all income received was reinvested back into the fund. The interpretation of the geometric mean is that it represents the compound annual return on investment for the four-year holding period 1997-2000.

IV. The Internal Rate of Return

A. Definition

The *internal rate of return* (IRR) is a well-known measure used extensively in capital budgeting applications. For investment management, the IRR is useful for computing returns when the amount of invested dollars does not stay constant over multiple periods. Using the data from Table I, suppose an investor invested \$10,000 in the fund at the beginning of 1997, invested another \$2,500 in the fund at the beginning of 1998, sold 380 shares at the beginning of 2000, and sold the remainder of the shares at the end of 2000.[4] Table II summarizes the investor's activities and the cash flows associated with the four-year holding period.

The correct method of computing the rate of return to the individual investor is to incorporate the intermittent cash flows into the analysis. The IRR allows us to include all cash flows for the specific period in which they took place. The IRR is the unique discount rate that satisfies (3).

$$\text{Initial Investment} = \Sigma\ [(NCF_n) / (1+IRR)^n] \qquad (3)$$

Most financial spreadsheet software and financial calculators will calculate the IRR for a specified cash flow stream. For the cash flows in Table II, the IRR is 14.39 percent.

Students find the explicit interpretation of the IRR difficult. As expressed in (3), the IRR is the discount rate that equates the present value of all future cash flows to the initial cost of investment. Alternatively, by moving the cost to the right-hand side of the equation, the IRR is the discount rate that makes the investment's net present value equal to zero. These definitions,

[3] See, for example, Bodie, Kane, and Marcus (2001, p. 155) and Levy (1999, p. 189).

[4] For simplicity, assume the closing end-of-year NAV from the previous year is the beginning NAV the first day of the subsequent year.

Table II: Intermittent Cash Flow Example

Date	Action	Number of Shares	Cumulative Number of Shares
1/1/97	Invests $10,000	547.95	547.95
1/1/98	Invests $2,500	109.89	657.84
1/1/00	Sells partial holding	380.00	277.84
1/1/01	Sells remaining holding	277.84	0

Summary of annual net cash flows to investor

Date	Action	Net Cash Flow
1/1/97	Purchased 547.95 shares @ $18.25	–$10,000.00
1/1/98	Purchased 109.89 shares @ $22.75 (–$2,500), Received dividends of $0.35 on 547.95 shares (+191.78)	–$2,308.22
1/1/99	No purchases or sales, Received dividends of $0.35 on 657.84 shares	$230.24
1/1/00	Received dividends of $0.50 on 657.84 shares (+$328.92) Sold 380 shares @ $26.35 (+$10,013)	$10,341.92
1/1/01	Dividends received of $0.50 on 277.84 shares (+$138.92) Sold 277.84 shares @ $29.90 (+$8,307.42)	$8,446.34

while theoretically correct, are not intuitive. Imagine an investor announcing that, when asked about the returns on his 401k, "the unique discount rate that makes the NPV of my fund zero is 15.05%!" This is a response that only a finance professor could love. Indeed, years of teaching business executives and requiring students to provide plain English interpretations of IRR solutions to textbook problems has convinced the authors the internal rate of return is the most used, and least understood, of the many tools taught in modern finance classes. Nevertheless, IRR is an important return measure of investment performance, because it allows the incorporation of intermittent cash flows that affect the rate of return.

B. The Reinvestment Assumption in IRR

One poorly understood aspect of the IRR is the implicit reinvestment assumption in the mathematics. The IRR represents the compounded periodic return on investment, assuming all intermittent cash flows over the investment period are reinvested at the explicit IRR. To see this, the cash flows from Table II are compounded forward to the end of the four-year holding period in Table III.

Table III: Illustration of IRR Reinvestment Assumption

End of Year Cash Flow	x	(IRR = 14.39%) Compound Factor	=	Value at End of Holding Period
–$2,308.22		$(1.1439)^3$		–$3,454.95
$230.24		$(1.1439)^2$		$301.27
$10,341.02		$(1.1439)^1$		$11,829.09
$8,446.34		$(1.1439)^0$		$8,446.34

Account Value at end of year 4 = $17,121.76

The sum of the reinvested dollars at the end of the fourth year is $17,121.76. Seen from this perspective, IRR answers the rather pertinent investment question: "If I invested $10,000 and four years later my account was worth $17,121.76, what rate of return on investment would I have earned?" The answer, as shown in Table III, is a compound annual rate of return over the four-year holding period of 14.39 percent.

C. The Problem of Multiple IRRs

Another characteristic of IRR that confounds students is the problem of multiple IRRs. If there is more than one change in the signs of the investment net cash flows (i.e., outflow, inflow, outflow, etc.) then there may be multiple solutions to the geometric mean of the product of the holding period returns.

Revisit the previous example used to illustrate IRR. Suppose we change the $2,500 additional investment from the beginning of 1998 to the beginning of 1999. All other aspects of the example remain the same. Table IV provides the investor actions and net cash flows for this four-year investment. Note that the signs of the first four net cash flows alternate from negative to positive. Such situations may produce more than one internal rate of return. To remedy this, students should assume an explicit reinvestment rate, compound all cash flows forward to the end of the investment life, and then find the growth rate that results in the initial investment growing into the expected total future dollars. The reinvestment rate assumption should be the investor's opportunity cost of capital, which in the "real world" is often the yield on a money market account into which brokerage dollars are automatically swept.

Assume a reinvestment rate of 3% annually. Compounding the net cash flows forward yields total future dollars of

$$\begin{aligned}
\$191.78(1.03)^3 &= \$209.56 \\
+\ \$\text{-}2308.22(1.03)^2 &= \$\text{-}2448.79 \\
+\ \$10{,}646.22(1.03)^1 &= \$10{,}656.61 \\
+\ \$8707.48(1.03)^0 &= \underline{\$8707.48} \\
&\quad \$17{,}124.86 \text{ total future dollars}
\end{aligned}$$

Table IV: Realized Compound Yield Example

Date	Action	Number of Shares	Cumulative Number of Shares
1/1/97	Invests $10,000	547.95	547.95
1/1/99	Invests $2,500	118.48	666.43
1/1/00	Sells partial holding	380.00	286.43
1/1/01	Sells remaining holding	286.43	0

Summary of annual net cash flows to investor

Date	Action	Net Cash Flow
1/1/97	Purchased 547.95 shares @ $18.25	–$10,000.00
1/1/98	No purchases or sales Received dividends of $0.35 on 547.95 shares (+191.78)	$191.78
1/1/99	Purchased 118.48 shares @ $21.10 (-$2,500), Received dividends of $0.35 on 547.95 shares	-$2308.22
1/1/00	Received dividends of $0.50 on 666.43 shares (+$333.22) Sold 380 shares @ $26.35 (+$10,013)	$10,346.22
1/1/01	Dividends received of $0.50 on 286.43 shares (+$143.22) Sold 286.43 shares @ $29.90 (+$8,564.26)	$8,707.48

Then, the compound annual return on investment is

$$(\$7{,}124.86 / \$10{,}000)^{1/4} - 1 = 14.39\%.$$

This compound annual return measure is known as realized compound yield in fixed-income analysis. For capital budgeting applications, it is the *Modified Internal Rate of Return*.

V. Reconciling the Geometric Mean Return With the Internal Rate of Return

The preceding analysis illustrates that the IRR is really just a special case of the geometric mean return.[5] The geometric mean assumes a constant dollar amount invested over the life of the

[5] Kosmicke and Opsal (1988, p. 19) note "An investor who forecasts returns by making long-run cash flow forecasts and then forcing out a discount rate that will equate the cash flow forecast with the current price of the stock will automatically be forecasting [geometric mean returns] . ."

investment, while the internal rate of return relaxes this assumption. Both, however, are measures of compound annual return on investment. This fact is often lost on both finance students and investors attempting to interpret measures of return. Two possible sources of the confusion lie within the calculation of IRR and the labels the finance profession has attached to each.

First, the IRR is rarely computed directly, since the mathematical procedure required is trial and error analysis to determine the specific rate that equates the present value of future dollars to the initial cost. As previously stated, explicit calculation is unnecessary, since the IRR function is built into spreadsheets and calculators. However, because the actual computations remain unseen, few students perceive the reinvestment assumption inherent in the mathematics, and thus the fact that IRR is a measure of compound periodic return is often overlooked. Barber and Odean (2000) discount the results of surveys of investment performance achieved by investment clubs, citing the Beardstown Ladies as an example of the difficulty investors have in accurately calculating multiperiod investment returns.

In addition, the investment management profession has given each measure a distinct label. The geometric mean return is known as a time-weighted measure of return; the internal rate of return is known as a dollar-weighted measure of return. Of course, the purpose for these labels is to acknowledge the fact that IRR allows for intermittent cash flows over the multiperiod investment horizon. Nevertheless, the result is that students and investors perceive the geometric mean and the IRR as completely different ways to measure multiperiod returns. As this analysis has shown, both are measures of compound periodic performance, or growth rates, over multiple investment holding periods.

VI. New Names for Compound Return Measures

A suggestion to alleviate confusion is to refer to the geometric mean return as the constant dollar compound rate of return, and the IRR as the nonconstant dollar compound rate of return. As previously detailed, the geometric mean is distinguished from the arithmetic mean by the fact that it incorporates compounding over the investment's life. Referring to it as a constant dollar compound rate of return retains this distinction.

Referring to IRR as the nonconstant dollar compound rate of return should allow students and investors to more easily comprehend that the IRR is really just a special case of the geometric mean return, allowing for intermittent cash flows. A precedent for this suggested change exists in the fact that the IRR of a bond is known as the yield to maturity. Changing the labels by which these return measures are known should clarify the fact that IRR is, in fact, a measure of compound annual return on investment.

VI. Summary

The purpose of this note is to clarify the assumptions inherent in the calculation of, and the relationship between, the geometric mean return and the internal rate of return. Both are important measures of multiperiod investment performance, yet the labels by which they are I dentified in investment management and the methods of calculation obscure the fact that each is a measure of compound periodic return. By understanding the reinvestment assumption inherent in the IRR, students and investors should perceive that the IRR is really just a special case of the geometric mean return. Using IRR to compute returns when intermittent cash flows occur over multiple periods allows investors to accurately incorporate the fact that a constant dollar amount is not invested over the entire holding period. Referring to IRR as a nonconstant dollar compound measure of return links it to geometric mean as a compound performance measure that is differentiated only by the relaxing of the constant dollar investment assumption. Together, these changes should allow better application and interpretation of these important measures of investment performance.

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