Extracting Implied Risk Aversion from Asset Prices and Other Economic Data

Travis L. Jones, Ph.D.*
Florida Gulf Coast University

Abstract

Since Pratt (1964) and Arrow (1971) presented the concept of risk aversion, many have attempted to calculate the “true” coefficient of risk aversion in the economy and have questioned how this measure reacts to changes in wealth and markets. Studies have proposed that risk aversion is constant, increasing, decreasing, and even U-shaped, across levels of wealth. This paper gives an overview of many studies that have extracted implied risk aversion measures from economic data and examines their assumptions and results. In addition, a simple model to compute risk aversion is presented, and the results are related to the level of the Standard & Poor’s (S&P) 500.

*The author would like to thank Catherine Chavez, for assistance with data collection, and James Ligon for his valuable comments.
1. Introduction

There are a number of studies that attempt to extract the level of implied risk aversion from economic data. These studies use two primary methods. The first method, examined primarily in the 1970s and 1980s and some in the 1990s, uses consumption data and returns from a stock market proxy, such as the S&P 500, to extract a coefficient of risk aversion, usually using a pre-specified utility function. The second method, developed more recently, involves extracting a risk aversion measure from options prices. This method calculates a measure of risk aversion using the subjective and risk-neutral probability densities of option prices and stems from the realization that option prices encompass a measure of risk that partially explains these prices.

This paper examines the literature that uses each of the primary methods of extracting implied risk aversion. The following section focuses on research that uses market data and stock returns to calculate a measure of risk aversion and provides the results of these studies. Section three examines literature that extracts risk aversion measures from options prices and presents the results of these studies. A simple model of risk aversion is then developed, and the results are related to the level of the S&P 500. The final section concludes.

2. Extracting Risk Aversion From Market Data

In his seminal paper on risk aversion, Arrow (1971) argues that preferences exhibit increasing relative risk aversion (IRRA) in wealth. This proposition says that if both a bet and a person’s wealth are increased by the same proportion, willingness to accept the bet will decrease. This hypothesis is far from intuitive, and Arrow agrees that it is “not so easily confrontable with intuitive evidence,” but defends the proposition with theoretical principles. Arrow does not empirically test his theory but uses likely bounds of the utility function to propose that the coefficient of relative risk aversion should be around one. The argument for IRRA rests on the proposition that as wealth increases, relative risk aversion cannot tend to a limit below one, and as wealth approaches zero, relative risk aversion cannot tend to a limit above one. These limits on relative risk aversion come from the fact that the utility function is bounded from above as wealth becomes infinite and bounded from below as wealth approaches zero.

Friend and Blume (1975) offer some criticism of Arrow’s proposal. They argue that the assumption that relative risk aversion increases in wealth is quite questionable. They argue that the actual test of this bounding assumption is to test the data empirically to discover the actual behavior of risk aversion. Their analysis is performed under the assumptions that capital markets are frictionless and that the market price of risk equals the value of all risky assets times a function of the coefficient of absolute risk aversion.

In their analysis, Friend and Blume calculate coefficients of relative risk aversion that range from 1.48 to 1.90, for a total wealth sample, and 1.47 to 2.14, for an income only sample. The fact that these figures vary somewhat is noted, and after adjusting for some sample bias, Friend and Blume report coefficients that range from 1.7 to 2.0. These results allow the authors to propose that the assumption of constant relative risk aversion (CRRA) is not a bad approximation for market preferences and that the coefficient of relative risk aversion is quite close to two and may possibly exceed two, a range that is approximately double the level that Arrow proposed.
While the evidence that Friend and Blume present demonstrates that relative risk aversion may be constant over wealth, the results of Hansen and Singleton (1982, 1984) provide evidence against preferences that exhibit constant risk aversion. Hansen and Singleton extract coefficients of relative risk aversion from stock returns and consumption that range from -1.26 to 1.16, using one measure of consumption, and 0.13 to 1.59, using another. The fact that both measures have a large range shows that the assumption of constant risk aversion may be incorrect.

Mehra and Prescott (1985) find the equity risk premium, the difference between the returns on the S&P 500 and riskless short-term debt, over the period from 1889 to 1978. Using the growth rate of per capita consumption, over the period, and a CRRA utility function, with the restriction that the coefficient of relative risk aversion not exceed 10, they find that, a coefficient of relative risk aversion of 55 was needed to generate the required curvature of the utility function in the model necessary to give the correct equity risk premium. The fact that this coefficient is so large led Mehra and Prescott to refer to this phenomenon as an equity premium puzzle. Even more astounding is that, according to Romer (2001), if the procedure of Mehra and Prescott was performed on the data from 1979 to 1999, the coefficient of relative risk aversion needed to satisfy the model would be 240!

Other studies find very different results for the coefficient of relative risk aversion. Cochrane and Hansen (1992) require a coefficient of relative risk aversion in the range of 40 to 50 to fit their model of consumption and equity risk premium. Grossman and Shiller (1981) use a CRRA utility function in a stock price-forecasting model to fit, what they call a “perfect foresight stock price,” to the actual consumption deflated S&P Composite Index, from 1889 to 1979, suggesting a CRRA coefficient of at least 4.0 to explain the variability of the stochastic process of the stock price.

3. Extracting Risk Aversion From Options Prices

The study by Bartunek and Chowdhury (1997) is one of the first of its kind to extract a measure of risk aversion from options prices. The authors solve for the coefficient of relative risk aversion by first simulating the value for a call option, using riskless probabilities. The “plug figure” that makes the simulated value of the call equal the observed market value is the coefficient of risk aversion. They use options prices on the S&P 100 index from January 1984 to December 1986, which result in coefficients of relative risk aversion, with significant t-statistics, in the range of 0.20 to 0.28. As Bartunek and Chowdhury note, this range is much less than most of the other studies, but it does fall within the range of Hansen and Singleton (1982, 1984). These coefficients seem to imply constant relative risk aversion, but this implication is most likely a result of the CRRA utility function chosen for the model.

The studies examined below use the same basic model for finding the coefficient of relative risk aversion from options prices. These studies take advantage of the relationship between the subjective probability and the risk-neutral probability of options prices. The difference in these papers is the methods used to obtain the probability densities and the additional contributions that each study makes. Even though the methodology is similar, each analysis finds a different measure for relative risk aversion, using different time periods and data.
Jackwerth (2000) uses the prices of the S&P 500 index options, S&P 500 index futures, and the actual S&P 500 stock index, from April 1986 through December 1995 and computes coefficient of absolute risk aversion for wealth levels close to one. These wealth levels allow this measure to be very comparable to the coefficients of relative risk aversion found in the other studies. Multiplying the absolute coefficient by the wealth level converts this measure to relative risk aversion.

Unlike other studies, Jackwerth divides his sample into different time periods. He does this primarily to determine if risk preferences seemed to change after the stock market crash of 1987. The range of the coefficient of absolute risk aversion from April 1986 to October 1987 is 0.0 to 4.8 (indicating relative risk aversion from about 0.0 to 4.6). The period from October 1987 to August 1993 exhibits characteristics where absolute risk aversion becomes U-shaped and ranges between −5.0 and 18 (relative risk aversion from about −4.8 to 18.9). Over the period from August 1993 to December 1995, the risk aversion function becomes even more U-shaped, is positive for lower wealth levels, becomes negative, and then becomes positive again. The coefficient of absolute risk aversion ranges from 14 to a low of −14 then increases to 19 (indicating relative risk aversion in the range of −13.9 to 19.9).

Aït-Sahalia and Lo (2000) examine the period from January 1993 to December 1993 and use the average bid and ask prices of S&P 500 index options and returns on the S&P 500 index. The values for the coefficient of relative risk aversion range from about 57 to a low of about 2, with an average at 12.7. The path of the coefficient, over the index value (or stock price), is U-shaped, and everywhere positive. This path implies a concave utility function, which is consistent with economic theory and also implies that the representative investor becomes more risk averse as the index goes down, or up, in value. Aït-Sahalia and Lo do not offer an explanation for this shape, other than to say that CRRA is misspecified, but this result seems to imply that investors may view both high and low levels of the S&P 500 with concern.

Coutant (2001) uses the CAC 40 index, of the Paris Stock Market, and CAC 40 options, traded on the MONEP, over the period from January 1995 to June 1997 to extract an implied risk aversion measure. The values calculated for relative risk aversion range from 1.05 to 11.40 and drop below 1, but do not go negative. Coutant comments that the level of risk aversion tends to be dependant on the evolution of the index over time and that the coefficient of relative risk aversion tends to respond to particular events or announcements involving the market. This reasoning seems to make economic sense, because uncertainty on the part of investors rises when major information is released.

Bliss and Panigirtzoglou (2001) use two different futures options contracts to extract coefficients of risk aversion. Data on the S&P 500 futures options range from February 1983 to June 2001, and data on the FTSE 100 futures options range from June 1992 to March 2001. In addition, the three-month LIBOR rates are used as a proxy for the riskless rate over the corresponding time periods. The power utility function gives CRRA coefficients on the S&P 500 options that range from 2.72 to 7.22 for horizons of 1 week to six weeks, respectively. The exponential utility function gives mean values for the coefficients on the S&P 500 options that range from 2.25 to 7.41 for horizons of 1 week to six weeks, respectively. For the FTSE 100 options, the power
utility function gives CRRA values in the range of 1.98 to 7.14, and the exponential utility function gives mean coefficient values from 1.73 to 6.27, both for horizons of 1 to six weeks, respectively. These results show that either utility function used returns similar coefficients of relative risk aversion, and also indicate that investors may be more risk averse the shorter their investment horizon.

The results of these studies, discussed above, are summarized in Table 1 and indicate a few of the differing views that have emerged on the behavior of the coefficient of risk aversion over wealth. These results depend greatly on the assumptions of the models and utility functions that produce them. The results of the pure consumption studies (Friend and Blume, 1975; Hansen and Singleton, 1982; 1984) indicate a somewhat well-behaved risk aversion in the aggregate, within a narrow range, compared to the other types of studies. The consumption and equity risk premium studies all support the equity premium puzzle first proposed by Mehra and Prescott (1985). These studies indicate excessively high, and quite non-intuitive levels of relative risk aversion. These coefficients can be interpreted to be too large to be used in describing the aggregate risk aversion level of the economy.

Table 1: Estimation of the coefficient of risk aversion from other studies

<table>
<thead>
<tr>
<th>Study</th>
<th>Data Used</th>
<th>Time Period</th>
<th>Coefficient Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrow (1971)</td>
<td>N/A</td>
<td>N/A</td>
<td>1</td>
</tr>
<tr>
<td>Friend and Blume (1975)</td>
<td>Total Wealth/Income</td>
<td>1962-1963</td>
<td>1.7 – 2.0</td>
</tr>
<tr>
<td>Bartunek and Chowdhury (1997)</td>
<td>S&amp;P 100 options</td>
<td>1984-1986</td>
<td>0.20 – 0.28</td>
</tr>
<tr>
<td>Jackwerth (2000)</td>
<td>S&amp;P 500 options</td>
<td>1986-1987</td>
<td>0.0 – 4.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1987-1993</td>
<td>-4.8 – 18.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1993-1995</td>
<td>-13.9 – 19.9</td>
</tr>
<tr>
<td>Coutant (2001)</td>
<td>CAC 40 options</td>
<td>1995-1997</td>
<td>0 – 11.4</td>
</tr>
<tr>
<td></td>
<td>FTSE100 options</td>
<td>1992-2001</td>
<td>1.73 – 7.14*</td>
</tr>
</tbody>
</table>

*Inclusive range, using two different data sets. †Inclusive range, using two different utility functions.

While they vary a great deal, the coefficients of relative risk aversion found from options prices provide some insight into investors’ preferences. The overall results from these studies are very intuitive when interpreted within financial theory. The results of Bliss and Panigirtzoglou (2001) indicate that no matter what option prices are used to find risk aversion, the representative investor seems to exhibit risk aversion within a similar range, based on the range of coefficients from S&P 500 and CAC 40 options. This study also indicates that investors are less risk averse for longer investment horizons, supporting the “invest for the long-term” proposition. From the results of Coutant (2001), investors appear to respond to major news releases and market events with increased risk aversion, corresponding to increased uncertainty. This result supports the results of Jackwerth (2000), which imply that investors’ risk aversion increased after the 1987 stock crash, also corresponding to increased uncertainty.
4. Risk Aversion Relative to the S&P 500

In an effort to link the results of the aforementioned studies to an easier to use model and to compare these results to an index, this study examines the equation used to calculate the coefficient of risk aversion presented in Bodie, et al. (2010). This equation allows novice market participants to compute a measure of risk aversion without the heavy mathematics of the studies noted above. The equation that Bodie, et al. present for the coefficient of risk aversion, $A$ is:

$$A = \frac{r_M - r_f}{\sigma_M^2},$$

(1)

where $r_M$ is the return on the market (i.e. S&P 500), $r_f$ is the risk-free rate (i.e. T-bills) and $\sigma_M^2$ is the variance of the market. Using monthly S&P 500 and T-bill data from January 1982 through December 2011, the return on the market is computed as the average of the monthly returns over the prior 60-months. The market variance is also computed over this same historical period. This methodology gives an overall average coefficient of 2.7, with a minimum of -5.2 and a maximum of 12.3 (see Table 2).

Following the approach of Jackwerth (2000), the data is examined from January 1982 through October 1987 as a pre-cash period. This pre-1987 crash period has an average risk aversion coefficient of 1.5, while the post-1987 crash period (Nov. 87-Dec. 11) has an average of 3.0. Given that two other market peaks occurred in August 2000 and September 2008, the risk aversion coefficient is examined over two additional periods. From November 1987 through August 2000, the average coefficient is 5.0. From September 2000 through September 2008, the average coefficient is 1.3, while from October 2008 through December 2011, the average coefficient is -0.4. Table 2 lists the average coefficients, along with the minimum and maximum, for these period, and Figure 1 graphs the coefficient of risk aversion against the S&P 500 over the full time examined.

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Average</th>
<th>Coefficient Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 1982-Dec. 2011</td>
<td>2.7</td>
<td>-5.2 – 12.3</td>
</tr>
<tr>
<td>Jan. 1982-Oct. 1987</td>
<td>1.5</td>
<td>-5.2 – 8.0</td>
</tr>
<tr>
<td>Nov. 1987-Aug. 2000</td>
<td>5.0</td>
<td>0.2 – 12.3</td>
</tr>
<tr>
<td>Sep. 2000-Sep. 2008</td>
<td>1.3</td>
<td>-2.9 – 10.4</td>
</tr>
<tr>
<td>Oct. 2008-Dec. 2011</td>
<td>-0.4</td>
<td>-4.1 – 0.9</td>
</tr>
</tbody>
</table>

Note: Risk aversion is measured using the model from Bodie, et al. (2010).
Comparing Table 1 to Table 2, it is evident that the coefficients from the simple model in Table 2, are most consistent with the findings of Jackwerth (2000) and Coutant (2001). The Bodie, et al. model produces coefficients in a range that is slightly higher than that of Coutant over the same period. The model results are also within the range of Jackwerth for all periods, with the single exception of July 1987. Figure 1 shows this peak of 8.0 at the end of July 1987, before the October 1987 crash. Thus, the results of this model are arguably as good as the results of the models that extract risk aversion from option prices. Yet, the model is much easier to use.

Figure 1 also demonstrates how risk aversion may change over time. As noted above, and demonstrated in Table 2 and Figure 1, the coefficient of risk aversion tends to move into different ranges, based on the time period examined. From mid-1985 through 2002, the coefficient remained positive. The coefficient began to fall in 1999, going negative in 2002. It then spiked in 2008, before the market crashed and the coefficient went negative again. From late-2009 though the end of 2011, the coefficient has remained around zero.

5. Conclusion

All of the above studies are performed under various assumptions. Many assume a utility function that requires risk aversion to behave a certain way. If these assumptions are relaxed, the
risk aversion coefficients may behave in a more intuitive manner. The results of these studies should be interpreted with caution, and the more information that is analyzed, the better the overall picture of the representative investor’s risk aversion.

The simpler model shows that average risk aversion may change over time and seems to be somewhat relative to the level of the market. Table 2 illustrates this change, showing that the average coefficient increases from the 1982 to 1987 period to the 1987 to 2000 period and coincides with an overall market increase during these periods. The average coefficient then decreases in the 2000 to 2008 and 2008 to 2011 periods, coinciding with a market decline, subsequent rally, and then another decline. However, the overall level of the S&P 500 was lower during these periods. Figure 1 shows how this measure somewhat follows the market and could be used (with caution) as a tool to examine the general level of the market.

Overall, the level of risk aversion in asset prices is an interesting statistic, however it is calculated. The results from calculating this statistic can be used to help interpret investors’ view of the market, in the aggregate, whether over a period of time or at a particular moment. This study has examined risk aversion from a preliminary view with the intent of comparing the results of complex and simple models for calculating risk aversion. The findings of this and previous studies show that the coefficient of risk aversion can fall within a wide range, depending on the model. Ultimately, it is up to the user to interpret the results relative to the overall output of the model being used.
References


