Stochastic Interest Rates and Short Maturity Currency Options

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Abstract

This paper uses a ten-year data set to examine the ability of the jump-diffusion models to explain systematic deviations in implicit distributions from the benchmark assumption of lognormality. Scott's (1997) calibrations found that stochastic interest rates should not affect short-maturity stock option prices much. Using transactions data from the Philadelphia stock exchange (PHLX) for European call and put currency options on the Deutschmark, the Japanese yen and sterling, over the period July 1984 to August 1989 and from March 1995 to December 1999, this study provides a robust proof that stochastic interest rates do affect short maturity currency options. The results are consistent with and incremental to Doffou and Hilliard (2002).

1. Introduction

Currency option pricing models premised on the Black-Scholes assumption of geometric Brownian motion exhibit severe specification error when fitted to market data. Systematic pricing errors, or equivalently, different implicit volatilities for different strike prices and maturities, are repeatedly found. Jumps do add to fat tails and jumps do provide likelihood to large changes in short time intervals. Most of recent attention has focused on the so-called "volatility smile", or U-shaped pattern in implicit volatilities for different strike prices; *e.g.,* Cao (1992) study of foreign currency options over the period 1987-89; Bollen and Raisel (2003) study of the alternative valuation models in the OTC currency option market; Carr and Wu (2004) study of the stochastic skew in currency options over the period 1996-2004; and Madan and Daal (2004) variance-gamma model for foreign currency options. The existence of specification error shows that the conditional distributions implicit in foreign currency option prices deviate substantially from the Black and Scholes benchmark assumption of lognormality.

Merton (1976) argued that a change in asset price can be split into two parts: a normal small change explained by a geometric Brownian motion, and an abnormal large change in price that can be modeled by a jump process. Indeed, the jump-diffusion stochastic process has been used to model large price changes that induce a fat-tailed distribution relative to the normal [see Jorion (1989), Ball and Torous (1985), Chacko and Das (2002), and Dupoyet (2004)].

Merton's (1976) jump-diffusion model allows for diversifiable jump risk. Scott (1997) results indicate that in a jump-diffusion model, the inclusion of stochastic interest rates adds little to the stock option prices. This is because jump risk is not priced by the market for stock options. When jump risk is systematic, general equilibrium models are necessary to derive option pricing formulas [see Bates(1991, 1996), Naik and Lee (1990), Perraudin and Sorensen (1994), and Dupoyet (2004)]. Bates' (1991, 1996) model permitted systematic jump exchange risk and derived the correct functional form of the market price of risk.

This study extends Doffou and Hilliard (2002). The idea is to verify if the result achieved in Doffou and Hilliard (2002) only based on a one-year data set can be sustained if the same methodology is applied to a large data set that spreads over a ten-year period. Hence, the objective of this paper is to test the robustness of the result in Doffou and Hilliard (2002). The out-of-sample performance of the jump-diffusion stochastic interest rate model is tested on a ten-year data set for pricing sterling, Deutschmark and Japanese yen currency futures options. The results are contrasted with those obtained by applying the same tests to Black's (1976) and Bates' models. In all three models tested, the results show that the jump-diffusion option pricing methodology explains the volatility smile and the strike price bias obtained in using Black's model based on a geometric Brownian motion assumption. Finally, the results show that stochastic interest rates do make a difference for short-maturity option prices. For example, as seen in Table B, for all Deutschmark call options, Black's model has a mean absolute percentage error of 22.63, Bates' has 7.79, and Doffou and Hilliard's has 4.71.

The paper is organized as follows. The next section describes the jump-diffusion stochastic interest rate model for pricing European currency futures options. The third section describes the data used. The estimation methodology is explained in the fourth section. The test results of pricing out-of-sample European currency futures options are presented in the fifth section, and the final section concludes the paper.

2. The Model

The jump-diffusion model used here is that developed by Doffou and Hilliard (2001), which can handle large price changes plus a smooth lognormal component. The focus is on options on a single short-term futures contract. Hence, the critical input to pricing short-term currency futures options is the assumption of a stochastic process for futures prices that generate a non-lognormal distribution. The domestic and foreign short interest rates, r_d and r_f, are assumed to follow the risk-neutralized stochastic processes

$$dr_d(\nu) = \left[f_\nu(t,\nu) + k_d f(t,\nu) + \frac{\sigma_d^2}{2k_d}\left(1 - e^{-2(\nu-t)k_d}\right) - k_d r_d(\nu) \right] dt + \sigma_d dZ_d \qquad (1)$$

$$dr_f(\nu) = \left[f_\nu^*(t,\nu) + k_f f^*(t,\nu) + \frac{\sigma_f^2}{2k_f}\left(1 - e^{-2(\nu-t)k_f}\right) - k_f r_f(\nu) \right] dt + \sigma_f dZ_f \qquad (2)$$

where dZ_d and dZ_f are increments to a standard Brownian motion, σ_d and σ_f are constant volatilities, and k_d and k_f are the speeds of adjustment related to the domestic and foreign interest rates, respectively. The terms $f(t,\nu)$ and $f^*(t,\nu)$ correspond to the instantaneous forward rate at time t for date $\nu > t$. The partial derivatives with respect to ν of $f(t,\nu)$ and $f^*(t,\nu)$ are $f_\nu(t,\nu)$ and $f_\nu^*(t,\nu)$, respectively.

The spot exchange rate (US$/foreign currency) follows a stochastic differential equation with (possibly asymmetric) random jumps:

$$dS/S^* = \left[r_d - r_f - \lambda^* k_m^*\right]dt + \sigma_S dZ_S + k^* dq^* \tag{3}$$

where Z_S is a standard Wiener process, σ_S is the instantaneous variance conditional on no jumps, and k^* is the risk-adjusted random percentage jump conditional on a Poisson-distributed event occurring, where $1+k^*$ is lognormally distributed:

$$\ln\left(1+k^*\right) \sim N\left[\ln\left(1+k_m^*\right) - 0.5\delta^2, \delta^2\right]$$

with $E\left(k^*\right) \equiv k_m^*$. The risk-adjusted frequency of Poisson events is λ^*, and q^* is a Poisson counter with intensity λ^*: $\Pr ob\left(dq^* = 1\right) = \lambda^* dt$ and $\Pr ob\left(dq^* = 0\right) = 1 - \lambda^* dt$. The spot exchange rate process is similar to the geometric Brownian motion process most of the time, but on average λ^* times per period the price jumps discretely by a random percentage. Jump random variables are uncorrelated, i.e, $\operatorname{cov}\left(dq^*, k^*\right) = 0$, $\operatorname{cov}\left(dZ_S, dq^*\right) = \operatorname{cov}\left(dZ_S, k^*\right) = 0$, $\operatorname{cov}\left(dZ_d, dq^*\right) = \operatorname{cov}\left(dZ_d, k^*\right) = 0$, and $\operatorname{cov}\left(dZ_f, dq^*\right) = \operatorname{cov}\left(dZ_f, k^*\right) = 0$.

The risk-neutralized process for the futures price is derived from Doffou and Hilliard (2001) as follows:

$$dF/F = -k_m^* \lambda^* dt + \sigma Z_F + k^* dq^* \tag{4}$$

where

$$\begin{aligned}\sigma^2\left(\sigma_S, \sigma_d, \sigma_f, \rho_{Sd}, \rho_{Sf}, \rho_{df}, k_d, k_f, \tau\right) = {} & \sigma_S^2 + \sigma_d^2 H_d^2(\tau) + \sigma_f^2 H_f^2(\tau) + 2\sigma_S \sigma_d \rho_{Sd} H_d(\tau) \\ & - 2\sigma_S \sigma_f \rho_{Sf} H_f(\tau) - 2\sigma_d \sigma_f \rho_{df} H_d(\tau) H_f(\tau)\end{aligned} \tag{5}$$

and $\quad H_d = \left(1 - e^{-tk_d}\right)/k_d \,,\quad H_f = \left(1 - e^{-tk_f}\right)/k_f$

and $\quad \tau = T - t\;$ is the time-to-maturity of the futures contract.

The solution of equations (1), (2), and (4) for European currency futures call / put option is a generalization of Bates' (1991) formula [see Doffou & Hilliard (2001)]. The values at time *t* for European call and put options that expire at time T_1 on a futures contract that expires at time *T* are respectively

$$c(t,T_1,T) = P(t,T_1)\sum_{n=0}^{\infty}\left[e^{-\lambda^*\tau_1}\left(\lambda^*\tau_1\right)^n / n!\right]\left[F(t,T)Z(t,T_1,T)e^{b(n)\tau_1}N(d_{1n}) - XN(d_{2n})\right] \tag{6}$$

$$p(t,T_1,T) = P(t,T_1)\sum_{n=0}^{\infty}\left[e^{-\lambda^*\tau_1}\left(\lambda^*\tau_1\right)^n / n!\right]\left[XN(-d_{2n}) - F(t,T)Z(t,T_1,T)e^{b(n)\tau_1}N(-d_{1n})\right] \tag{7}$$

where
$$b(n) = -\lambda^* k_m^* + n\gamma^* / \tau_1$$

$$d_{1n} = \left[\ln\{F(t,T)Z(t,T_1,T)/X\} + b(n)\tau_1 + \left(\nu^2 + n\delta^2\right)/2\right] / \left(\nu^2 + n\delta^2\right)^{\frac{1}{2}}$$

$$d_{2n} = d_{1n} - \left(\nu^2 + n\delta^2\right)^{\frac{1}{2}}$$

and
$$Z(t,T_1,T) = \exp\left[-\sigma_d\sigma_S\rho_{dS}\left(\tau_1 - H_d(\tau_1)\right)/k_d\right]$$
$$\times \exp\left[-\frac{\sigma_d^2}{k_d^2}\left(\tau_1 - H_d(\tau_1) - \left(e^{-(T-T_1)k_d} - e^{-\tau k_d}\right)/k_d + \left(e^{-\tau_1 k_d} - e^{2tk_d - (T+T_1)k_d}\right)/2k_d\right)\right]$$
$$\times \exp\left[-\frac{\sigma_d\sigma_f\rho_{df}}{k_d k_f}\left\{\tau_1 - H_d(\tau_1) - \frac{e^{-(T-T_1)k_f} - e^{-\tau k_f}}{k_f} + \frac{e^{-\tau_1 k_f} - e^{t(k_d + k_f) - (T+T_1)k_d}}{k_d + k_f}\right\}\right].$$

The scaling factor Z(t, T_1, T) shifts the mean of the implicit distribution because of exchange rate and interest rate correlations. Option pricing at the expected discounted terminal payout is not the same as the discounted expected terminal payout under stochastic interest rates. This scaling factor explains the key empirical difference between this general model and Bates (1991, 1996) model. It takes the value of one when the domestic interest rate is constant ($\sigma_d = 0$). Another difference between this general model and Bates' model is the term structure of implied volatilities which is not explored here because only one option maturity is considered at any given time. The values of the scaling factor under stochastic interest rates are provided in Doffou and Hilliard (2002), table 1, page 52. The detailed computation of the integral of the futures return variance, $\nu^2(t, T_1, T)$, over the life of the option is given in Doffou and Hilliard (2001), equations (23) and (24). When ν^2 is replaced by $\sigma_S^2\tau_1$ and Z(t, T_1, T) = 1, special cases of equations (6) and (7) give the Bates (1991, 1996) futures call and put formulas. When $k_m^* = \lambda^* = \delta = 0$, special cases of equations (6) and (7) give the Black (1976) call and put formulas. The price at time t of the domestic zero-coupon bond maturing at time T is $P(t,T)$.

Bates' and Black's formulas use the spot exchange rate volatility as a proxy for futures price volatility and therefore assume the term structure of futures volatility to be flat. The proposed model (Doffou and Hilliard's model) adequately predicts the difference between spot and futures volatility at different futures maturities. The futures call and put option prices given in equations (6) and (7) do not depend directly on the three state variables of the model, but they depend indirectly on these state variables through the level of the exogenous futures price $F(t, T)$.

The mean jump size, k_m^*, affects the skewness, while the jump frequency parameter λ^* shapes mostly the kurtosis of the distribution of futures price generated by the diffusion equation (4). A positive/negative mean jump size creates a fatter right/left tail in the distribution. In Merton (1976b) and subsequent applications of that model such as Ball and Torous (1985), the value of the mean jump size is set at zero, assuming a symmetric distribution. In this paper, the mean jump size parameter k_m^* takes different non-zero values necessary to explain the asymmetric (skewed) distribution of currency futures prices. Different values of the average jump size k_m^* and the jump frequency parameter λ^* generate various shapes for the distribution of futures prices as in Das and Baz (1996). The proposed model appears robust enough to explain most pricing bias generated when using the Bates' or Black's model.

To run the empirical tests, the risk-neutralized jump-diffusion parameter vector, $\beta = [\sigma, k_m^*, \lambda^*, \delta]$, and the exogenous variables to the futures call and put option pricing formula are needed. These exogenous variables are: the current futures price $F(t, T)$, the option strike price X, the domestic interest rate r_d, the domestic bond price $P(t, T_1)$, the time-to-maturity of the option $\tau_1 = T_1 - t$, the time-to-maturity of the futures contract $\tau = T - t$, the expiration dates of the option and the futures contract, T_1 and T, respectively, and the current time t.

3. The Data

Transactions data from the Philadelphia stock exchange (PHLX) for European call and put currency options on the Deutschmark, the Japanese yen and the sterling are used. The data consist of the time and price of every transaction in which the price changed from the previous transaction for the period July 1984 to August 1989 and from March 1995 to December 1999. Because the out-of-sample performance of the proposed model is tested here, the model parameters must be estimated using a data set that is outside the time period of the data set used to test the model. The model parameters are estimated in Doffou and Hilliard (2002) using monthly exchange rate and short term interest rate data from September 1989 to February 1995.

Currency futures contracts are available for March, June, September, and December expiration dates, with delivery taking place on the third Wednesday of the month. The last trading day is the third business day before delivery. Options are traded on all four contracts. Bid and ask data were discarded since no transactions were conducted at those prices. Only contracts of a single maturity are considered for any day: namely, quarterly contracts with maturities between one and four months. Longer maturities were too thinly traded, and shorter maturities were too close to the maturity date to contain enough information about implicit distributions. To avoid days with thin trading, at least 20 call and 20 put transactions are required for any given day's data to be retained. Transactions in at least four strike classes for calls and four for puts are required to ensure a range of "moneyness" sufficient to provide a good picture of the underlying distribution. Deep-in-the-money and deep-out-of-the-money call and put options, defined as those with prices less than 5 cents, are deleted from the sample. For these options, the bid-ask spread is a big proportion of their time value. Therefore, their implied volatilities would be highly sensitive to the bid-ask bounce.

For each option, its specific underlying futures contract is used. The nearest futures transaction of comparable maturity preceding each option transaction is used as the underlying futures price. Currency futures contracts maturing close to concurrently with the PHLX European options on currency spot and traded on the International Monetary Market (IMM) at the Chicago Mercantile Exchange (CME) are used. This market is liquid and the exchange keeps a record of all transactions, called the "IMM Statistics Department Quote Capture Report". Although it is not possible to match a PHLX European option on currency spot trade with a trade on the corresponding PHLX currency contract, it is possible to closely match this option on currency spot trade with a currency futures price quotation on the IMM.

To synchronize the data, only those pairs of option/futures ranging less than five minutes apart are used. The time lapse between the futures and options transactions averages about 30 seconds for Deutschmark, Japanese yen and sterling options. Outliers are not purged from the sample, consistent with the assumption of a jump process.
All of the above criteria eliminate 213 out of 1908 trading days for Deutschmark futures options, 545 out of 1724 trading days for Japanese yen futures options, and 307 out of 1952 trading days for sterling futures options. For all three currencies and 40 contracts considered, 4519 days are selected for parameter estimation. The daily risk-free interest rate is computed from the prices of Treasury bills maturing close to the maturity of the option. The sample data decomposition is provided in Table A.

4. Estimation

4.1 Estimation of parameters not related to the jump-diffusion process

The estimation methodology mimics the one in Doffou and Hilliard (2002). The parameters of interest are the domestic short interest rate term structure inputs k_d and σ_d, the foreign short interest rate term structure inputs k_f and σ_f, the coefficient of correlation between the domestic and foreign short rates, ρ_{df}, the coefficient of correlation between the spot exchange rate and the domestic short rate, ρ_{sd}, and the coefficient of correlation between the spot exchange rate and the foreign short interest rate, ρ_{sf}.

To estimate these parameters, monthly exchange rate data are taken from the *International Financial Statistics* for 65 observations from September 1989 to February 1995. The annualized domestic (US) and foreign short-term interest rate series, r_d and r_f, are taken from the *International Currency Review*. The exchange rates, S, are in dollars per unit of foreign currency. Treasury bill rates are used for the US and Britain; call money rates are used for Germany and Japan.

The spot domestic and foreign interest rates for three, six, and twelve months are taken from *The Financial Times* because they were not available in the *International Currency Review*. The six-month forward rates starting at the end of month six, and the nine-month forward rates starting at the end of month three, are computed using these observable domestic and foreign spot interest rates. All the aforementioned parameters are computed for the Deutschmark, sterling, and the Japanese yen. The speeds of adjustment, K, are estimated every day from daily term structures. The results of the non-jump-related parameters estimates are provided in Doffou and Hilliard (2002), Tables 3 and 4. The covariance matrices of the spot exchange rate, the domestic interest rate, and the foreign interest rates, for each of the currencies considered, are available in Doffou and Hilliard (2002), Table 3, page 56. Estimates for the coefficients of correlation, ρ_{df}, ρ_{ds}, and ρ_{sf}, are derived from this same Table 3, page 56. The values of the speeds of adjustment, K_d and K_f, the interest rate volatilities, σ_d and σ_f, are provided in Doffou and Hilliard (2002), Table 4, page 57, and are estimated by solving the generalized Vasicek bond pricing equation. The estimation technique used is consistent with equations (1), (2), and (3), and is explained in Doffou and Hilliard (2002), at the bottom of Table 4, page 57. The speeds of adjustment are solved using a non-linear system of equations module available in Gauss. The exchange rate and interest rate parameters change daily. Hence, they are estimated periodically (daily, weekly, or monthly) to test the model. The interest rate process parameters in Doffou and Hilliard (2002), Table 4, Page 57, are estimated daily.

4.2 Estimation of the jump-diffusion parameters

The estimation method used here is the one described in Doffou and Hilliard (2002). The method consists in implying the jump-diffusion parameters using the option pricing formula and transaction data on futures and futures options. The proposed three-state model requires four parameters as inputs. The parameters of the model can be estimated even by using a short time series of past data. Basically, the estimation can be done with data from any interval which has a sufficient number of trades. The implied estimation technique eliminates partially the problem of preferences, requires no further assumption about utility function, and gives the risk-neutralized version of these parameters. The technique for extracting the parameters of the model parallels that used for extracting implied volatility with Black's model. The four parameters of the risk-neutralized process are implied by minimizing the sum of the squared errors of all options in the sample as follows

$$S(\beta) = \sum_{t=1}^{n_t} \sum_{i=1}^{n_i} \left[\varepsilon_{i,t}(\beta)\right]^2 \tag{8}$$

where $\varepsilon_{i,t}$ is the pricing error for each option given by

$$\varepsilon_{i,t} = V_{i,t} - V(F_{i,t}, X_{i,t}, \tau_t, r_t; \beta), \quad i = 1, \ldots, n_i; \quad t = 1, \ldots, n_t \tag{9}$$

where $V_{i,t}$ is the ith observation of the market-observed futures option price on day t, $\beta = \left[\sigma, k_m^*, \lambda^*, \delta\right]$ is the vector of unknown parameters in the model, and $\beta = \sigma$ is the volatility parameter in Black's model. The vector of unknown parameters for the proposed model has the same appearance as that in Bates' model except that k_m^*, λ^*, and δ have different values due to the fact that the scaling factor $Z(t, T_1, T)$ is not constrained to one as is the case in Bates' model. The $V(.)$ function is given by the proposed model, then by Bates' and Black's option pricing formula. The number of trading days in the estimation sample used is n_t, and n_i represents the number of observations in the sample per trading day. The performance of the models as asset pricing tools is tested using only out-of-sample data.

5. Out-of-sample pricing performance

5.1 Tools of performance measurement

An option pricing model out-of-sample performance is measured by the stability of the parameters and the accuracy of the parameter estimates. A test of the out-of-sample performance of an option pricing model is a test of how slowly the parameters change over time. The pricing biases of the proposed model (Doffou and Hilliard's model) are compared to those generated by Bates' and Black's models using the mean relative percentage option pricing error (MRE) and the mean absolute percentage option pricing error (MARE) as in Doffou and Hilliard (2002). These statistics are computed below:

$$MRE = \frac{1}{n}\sum_{i=1}^{n}\left(V_i^* - V_i\right)/V_i \tag{10}$$

$$MARE = \frac{1}{n}\sum_{i=1}^{n}\left(\left|V_i^* - V_i\right|\right)/V_i \tag{11}$$

where the price of the option observable in the market is V_i, and the option price given by the model is V_i^*. The number of options in an option class characterized by moneyness, time-to-maturity, estimation delay, and contract month is n. The estimation delay is the number of days the model can be used before its parameters can be re-estimated. The *MRE* statistic is more appropriate for testing model pricing bias for a specific option class. If the model prices a class of option accurately, the *MRE* will converge to zero in that class as the number of options increases.

A regression analysis is used to investigate the effects of moneyness and maturity on pricing bias. The regression equation is

$$PE_i = \alpha_1 + \alpha_2\left(F/X\right)_i + \alpha_3\left(F/X\right)_i^2 + \alpha_4\tau_i + \alpha_5\tau_i^2 + \varepsilon_i \ , \ i = 1,.........,n \tag{12}$$

where $PE_i = V_i^* - V_i$ is the dependent variable, a measure of the option pricing error for each observation, and given by the difference between the model price and the market price. The independent variables are the moneyness, *F/X*, and the time-to-maturity, τ. The squared terms of the moneyness and the time-to-maturity are included in the regression to examine non-linear relationships. The regression is run over the entire sample but separately for calls and puts. This is a cross-sectional regression. Therefore, the standard errors are computed using the White heteroscedasticity consistent estimator.

5.2 Results

The mean absolute percentage pricing errors (*MARE*) for futures options on the Deutschmark, sterling, and Japanese yen by moneyness and time-to-expiration, expressed in days are shown in Tables B, C, and D. This statistics is computed for the proposed three-state model as well as for Bates' and Black's models. The model parameters are estimated with data from Mondays and tests are performed on Tuesday's prices, with a one-day lag. The results are consistent with Doffou and Hilliard (2002). For all classes of options, regardless of the moneyness and the time-to-expiration, the three-state model performs better than Bates' model which in turn performs better than Black's model, for all three currencies examined. Using the *MRE* statistics leads to the same conclusion. For the proposed model and Bates' model, the larger errors occur for out-of-the-money options because options in the tails of the distribution are more sensitive to the non-stationarity of the distribution. This is an indication that the distributions for currency futures options are positively skewed. This finding contrasts recent empirical work on S&P500 Index options indicating that the implied distribution of the S&P500 Index is more negatively skewed than the lognormal.

Table E shows the mean absolute percentage pricing errors for futures options by moneyness and estimation delay, in days, for the Deutschmark. The estimation delay is introduced to test the stability of the parameter estimates within the trading week. The *MARE* are computed for the proposed three-state model as well as for Bates' and Black's models. For all estimation lags and strike price classes, the jump-diffusion stochastic interest rates model or three-state model performs far better than Bates' model, which in turn performs better than Black's model. For the proposed model, the larger errors occur with out-of-the-money calls and out-of-the-money puts. The numbers show that the in-the-money options are least sensitive to changes in the parameters of the underlying price process. For all three currencies considered, the *MARE* are greater for the four-day estimation lag than for the one-day estimation lag for deep-in-the-money call options ($F/X > 1.05$) and for deep-in-the-money put options ($F/X < 0.95$). These pricing errors computed are conditional on the realized exchange rate one to four days hence.

The results of the cross-sectional regressions of the pricing errors for Deutschmark European futures options appear in Table F. The R^2 or coefficient of determination of the regressions for both the three-state model and bates' model is very small. Beside a very low explanatory power supported by a very low R^2, the F-statistic for both Bates' and the three-state models is low. To be statistically significant, a regression equation coefficient must be statistically significant for both call and put. Hence, systematic pricing errors relative to moneyness and time-to-maturity have been eliminated from pricing errors in these models. For Black's model, the time-to-maturity regression coefficients are not statistically significant while the moneyness coefficients are statistically significant. Moreover, the F-statistic for Black's model is very high. Therefore, moneyness is a good explanatory variable for pricing bias in Black's model, while time-to-maturity is not. These results are the same for all three currencies examined and are consistent with Doffou and Hilliard (2002).

6. Conclusions

This work analyzes the out-of-sample performance of Black's (1976), Bates' (1991, 1996) and Doffou and Hilliard (2001) models for pricing European currency futures options in a Deutschmark, sterling and Japanese yen futures market, using a ten-year data set. Black's model systematically misprices options in the sample examined. Doffou and Hilliard (2001) model outperforms both Bates' and Black's models, and Bates' model performs better than Black's model. The results show that the performance improves for both Doffou and Hilliard's model and Bates' model when the parameters are re-estimated daily. These two models incorporate jumps in the spot exchange rate and have parameters that are not stationary over time.

The jump frequency parameter, λ^*, and the mean jump size parameter, k_m^*, are estimated but not reported here, and are persistently positive for all estimation days and for all three currencies examined. Therefore, the US$/Deutschmark, US$/sterling and US$/Japanese yen skewness premium is positive. This positive skewness premium is explained by the correlation between the exchange rate and volatility shocks, and is an indication of a relative strength of the dollar in the period examined. These results confirm that the distribution of changes in futures prices is more positively skewed and more leptokurtic than the normal distribution. This explains why Black's model underprices out-of-the-money puts and overprices out-of-the-money calls.

The results are incremental to Doffou and Hilliard (2002) and are a robust proof that stochastic interest rates strongly affect short-maturity currency option prices. Scott (1997) results indicate that in a jump-diffusion model, the inclusion of stochastic interest rates adds little to stock option prices. This, because Merton (1976) proves that jump risk is unsystematic for stock options. Hence, jump risk for stock options is diversifiable, that is not priced by the market. The intuition behind this is that positive jumps and negative jumps cancel out over time for stock options. This is not the case for currency options. Bates (1991, 1996) shows that jump risk is not diversifiable for currency options and derives the correct functional form of the market price of risk. Hence, jump risk is systematic for currency options and does matter. Because jumps do not cancel out over time for currency options, they are affected by stochastic interest rates. The results also show that the random volatility inherent in the Bates model is not as important as the random interest rates in the proposed model for currency options. Doffou and Hilliard (2001) model provides the skewness and kurtosis required to fit the empirical distribution better than Bates' model and far better than Black's model.

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Table A: Analysis Of The Sample

Contract	Days			Mean number of matched calls			Mean number of matched puts			Mean number of sample calls			Mean number of sample puts		
	DM	£	¥	DM	£	¥	DM	£	¥	DM	£	¥	DM	£	¥
Sept 84-88	398		247	1121.2		969.8	381.1		302.5	227.3		181.1	146.4		105.3
95-99		379			1108.5			379.3			233.6			140.1	
Dec 84-88	302		294	823.7		648.4	342.2		158.8	357.1		293.8	163.9		109.1
95-99		362			757.3			312.8			368.2			170.6	
Mar 85-89	437		197	741.4		687.1	351.9		333.6	229.2		197.7	117.6		94.7
95-99		394			729.6			355.3			211.5			105.1	
June 85-89	558		441	1368.6		1196.6	653.2		547.9	526.6		365.3	297.7		186.2
95-99		510			1348.4			643.4			517.1			283.5	
All	1695		1179	1013.7		875.4	432.1		335.7	335.1		259.5	181.4		123.8
		1645			985.9			422.7			332.6			174.8	

The number of days for which the sample was large enough for estimation (at least 20 call and 20 put transactions) is represented by “Days”. The mean for each contract of the daily number of calls and puts matched to an underlying futures transaction are called “matched calls” and “matched puts”. The mean for each contract of the daily number of calls and puts used for estimating equation (9) are “sample calls” and “sample puts”. The total for the days column and the average for the other columns are shown in the last row.

Table B: Deutschmark Pricing Errors By Moneyness And Time-To-Maturity

	D & H model		Bates' model		Black's model	
	Calls	Puts	Calls	Puts	Calls	Puts .
$\tau \leq 30$						
F/X ≤ 0.950	7.06	4.62	9.11	5.33	29.41	7.21
0.95 < F/X ≤ 0.980	6.43	4.45	7.39	5.12	13.32	7.17
0.98 < F/X ≤ 1.020	5.31	5.19	6.69	6.18	11.13	8.79
1.02 < F/X ≤ 1.050	4.28	6.33	6.47	7.51	11.71	14.56
1.05 < F/X	3.52	9.38	5.69	10.43	11.37	27.11
All *F/X*	4.38	6.12	8.09	7.67	18.24	14.13
$30 < \tau \leq 60.0$						
F/X ≤ 0.950	7.27	4.38	9.24	5.31	33.35	9.46
0*.95 < F/X ≤ 0.980*	6.39	4.17	7.12	5.22	17.48	9.19
0.98 < F/X ≤ 1.020	5.28	4.72	6.44	5.69	15.33	10.22
1.02 < F/X ≤ 1.050	3.66	6.17	5.21	7.28	15.51	16.13
1.05 < F/X	3.49	9.11	5.62	10.14	14.86	29.09
All *F/X*	4.72	5.55	7.73	6.81	23.17	16.11
$60 < \tau \leq 90.0$						
F/X ≤ 0.950	8.12	5.69	10.41	6.37	33.09	13.59
0.95 < F/X ≤ 0.980	6.61	5.53	8.19	6.13	17.11	13.27
0.98 < F/X ≤ 1.020	5.93	5.71	7.64	6.38	15.27	14.33
1.02 < F/X ≤ 1.050	5.08	7.25	7.13	8.21	15.47	20.61
1.05 < F/X	4.24	10.39	6.67	11.11	15.21	33.39
All *F/X*	6.31	7.17	8.79	8.48	23.39	20.22
$90 < \tau$						
F/X ≤ 0.950	6.49	4.51	8.81	5.27	33.28	12.58
0.95 < F/X ≤ 0.980	4.37	4.28	6.53	4.92	17.19	12.17
0.98 < F/X ≤ 1.020	4.88	4.93	6.14	5.68	14.79	13.79
1.02 < F/X ≤ 1.050	3.49	6.27	5.33	7.39	15.26	19.43
1.05 < F/X	3.13	8.35	4.96	10.21	14.91	32.31
All *F/X*	4.37	5.12	7.27	6.81	22.98	19.08
All τ						
F/X ≤ 0.950	7.54	4.65	9.48	5.61	33.13	11.71
0.95 < F/X ≤ 0.980	5.22	4.31	7.25	5.42	17.23	10.83
0.98 < F/X ≤ 1.020	5.14	4.72	6.66	6.21	14.77	12.19
1.02 < F/X ≤ 1.050	4.33	6.29	6.17	7.82	14.96	18.28
1.050 < F/X	3.82	8.40	5.89	10.91	14.73	30.37
All *F/X*	4.71	5.53	7.79	7.49	22.63	18.21

D & H model stands for Doffou and Hilliard jump-diffusion stochastic interest rates model. Results are mean absolute pricing errors for PHLX Deutschmark European futures options by moneyness, *F/X*, and time-to-maturity, τ, in days. Transaction periods are 7/1/84 to 8/31/89 and 3/1/95 to 12/30/99. All 40 contracts are included. Data are estimated on Mondays and the models tested on Tuesdays.

Table C: Sterling Pricing Errors By Moneyness And Time-To-Maturity.

	D & H model		Bates' model		Black's model	
	Calls	**Puts**	**Calls**	**Puts**	**Calls**	**Puts**
$\tau \leq 30$						
F/X ≤ 0.950	8.12	5.34	10.28	6.53	30.09	8.63
0.95 < F/X ≤ 0.980	7.59	5.21	8.68	6.61	14.53	8.27
0.98 < F/X ≤ 1.020	6.33	6.13	7.74	7.31	12.42	9.69
1.02 < F/X ≤ 1.050	5.17	7.29	7.38	8.79	12.47	15.27
1.05 < F/X	4.20	10.61	6.46	11.45	12.91	28.04
All *F/X*	5.39	7.38	9.23	8.81	19.87	15.35
$30 < \tau \leq 60.0$						
F/X ≤ 0.950	8.21	5.48	10.37	6.33	34.52	10.69
0.95 < F/X ≤ 0.980	7.46	5.17	8.19	6.11	18.53	10.37
0.98 < F/X ≤ 1.020	6.39	5.44	7.58	6.66	16.19	11.58
1.02 < F/X ≤ 1.050	4.79	7.27	6.83	8.31	16.53	17.42
1.05 < F/X	4.51	10.22	6.71	11.23	15.68	30.19
All *F/X*	5.72	6.78	8.49	7.92	24.22	17.51
$60 < \tau \leq 90.0$						
F/X ≤ 0.950	9.28	6.49	11.51	7.73	34.15	14.82
0.95 < F/X ≤ 0.980	7.34	6.23	9.19	7.40	18.29	14.49
0.98 < F/X ≤ 1.020	6.49	6.74	8.58	7.93	16.34	15.77
1.02 < F/X ≤ 1.050	6.04	8.18	8.07	9.71	16.11	21.81
1.050 < F/X	5.17	11.39	7.41	12.39	16.07	34.50
All *F/X*	7.23	8.19	9.68	9.25	24.61	21.67
$90 < \tau$						
F/X ≤ 0.950	7.43	5.39	9.62	6.54	34.28	13.69
0.95 < F/X ≤ 0.980	5.28	5.06	7.37	5.08	18.11	13.18
0.98 < F/X ≤ 1.020	5.71	5.69	7.09	6.27	15.10	14.31
1.02 < F/X ≤ 1.050	4.58	7.13	6.14	8.68	16.57	20.48
1.05 < F/X	4.19	9.29	5.15	11.37	15.12	33.27
All *F/X*	5.68	6.17	8.59	7.27	23.13	20.18
All τ						
F/X ≤ 0.950	8.21	5.69	10.08	6.59	34.28	12.21
0.95 < F/X ≤ 0.980	6.73	5.17	8.38	6.27	18.14	11.53
0.98 < F/X ≤ 1.020	6.17	5.58	7.54	7.16	15.39	13.69
1.02 < F/X ≤ 1.050	5.21	7.47	7.08	8.61	15.83	19.31
1.05 < F/X	4.11	9.09	6.68	11.27	15.33	31.72
All *F/X*	5.35	6.52	8.33	8.21	23.23	19.31

D & H model stands for Doffou and Hilliard jump-diffusion stochastic interest rates model. Results are mean absolute percentage pricing errors for PHLX sterling European futures options by moneyness, *F/X*, and time-to-maturity, τ, in days. Transaction periods are 7/1/84 to 8/31/89 and 3/1/95 to 12/30/99. All 40 contracts are included. Data are estimated on Mondays and the models tested on Tuesdays.

Table D: Japanese Yen Pricing Errors By Moneyness And Time-To-Maturity.

	D & H model		Bates' model		Black's model	
	Calls	**Puts**	**Calls**	**Puts**	**Calls**	**Puts**
$\tau \le 30$						
$F/X \le 0.950$	9.18	6.69	11.19	7.28	31.11	9.71
$0.95 < F/X \le 0.98$	8.31	6.28	9.69	7.18	15.61	9.11
$0.98 < F/X \le 1.020$	7.04	7.01	8.57	8.24	13.21	10.26
$1.02 < F/X \le 1.050$	6.49	8.33	8.20	9.70	13.41	16.62
$1.05 < F/X$	5.68	11.02	7.79	12.11	13.82	29.72
All F/X	6.53	8.13	10.06	9.23	21.15	16.17
$30 < \tau \le 60.0$						
$F/X \le 0.950$	9.11	6.49	11.12	7.21	35.62	11.48
$0.95 < F/X \le 0.980$	8.15	6.13	9.51	7.13	19.81	10.99
$0.98 < F/X \le 1.020$	7.04	6.69	8.18	7.79	17.33	12.64
$1.02 < F/X \le 1.050$	5.61	8.08	7.59	9.41	17.71	18.93
$1.05 < F/X$	5.43	10.56	7.09	12.15	16.21	31.52
All F/X	6.73	7.43	9.28	8.64	25.52	18.49
$60 < \tau \le 90.0$						
$F/X \le 0.950$	10.63	7.51	12.13	8.23	35.82	15.42
$0.95 < F/X \le 0.980$	8.27	7.39	10.41	8.10	19.91	15.13
$0.98 < F/X \le 1.020$	7.38	7.71	9.68	8.79	17.73	16.29
$1.02 < F/X \le 1.050$	7.03	9.36	9.14	10.58	17.97	22.78
$1.05 < F/X$	6.18	12.51	8.70	13.36	17.58	35.61
All F/X	8.01	9.13	10.11	10.61	25.81	22.13
$90 < \tau$						
$F/X \le 0.950$	8.63	6.28	10.13	7.31	35.39	14.82
$0.95 < F/X \le 0.980$	6.33	6.09	8.68	6.22	19.22	14.53
$0.98 < F/X \le 1.020$	6.69	7.01	8.27	7.81	16.61	15.89
$1.02 < F/X \le 1.050$	5.58	8.27	7.51	9.63	17.21	21.12
$1.05 < F/X$	5.48	10.22	6.31	12.33	16.41	34.87
All F/X	6.58	7.29	9.41	8.83	24.71	21.79
All τ						
$F/X \le 0.950$	9.58	6.29	11.12	7.85	35.23	13.77
$0.95 < F/X \le 0.980$	7.38	6.01	9.49	7.51	19.12	12.25
$0.98 < F/X \le 1.020$	7.10	6.18	8.58	8.63	16.21	14.44
$1.02 < F/X \le 1.050$	6.23	8.17	8.21	9.81	16.43	20.22
$1.05 \le F/X$	5.15	10.33	7.31	12.74	16.18	32.78
All F/X	6.69	7.67	9.79	10.11	24.19	20.68

D & H model stands for Doffou and Hilliard jump-diffusion stochastic interest rates model. Results are mean absolute percentage pricing errors for PHLX Japanese yen European futures options by moneyness, *F/X*, and time-to-maturity, τ, in days. Transaction periods are 7/1/84 to 8/31/89 and 3/1/95 to 12/30/99. All 40 contracts are included. Data are estimated on Mondays and the models tested on Tuesdays.

Table E: Deutschmark Pricing Errors By Moneyness And Estimation Delay.

	D & H model		Bates' model		Black's model	
	Calls	Puts	Calls	Puts	Calls	Puts .
One-day estimation delay						
$F/X \leq 0.950$	9.02	3.27	11.22	3.48	41.11	4.96
$0.95 < F/X \leq 0.980$	5.39	3.09	6.56	3.37	11.83	3.78
$0.98 < F/X \leq 1.020$	4.52	4.17	5.49	4.72	6.78	6.84
$1.02 < F/X \leq 1.050$	3.69	6.65	4.15	7.79	6.99	19.22
$1.05 < F/X$	3.10	11.34	3.37	13.49	6.18	41.37
Two-day estimation delay						
$F/X \leq 0.950$	14.19	3.31	15.71	3.58	41.31	4.92
$0.95 < F/X \leq 0.980$	7.28	4.04	8.81	4.33	10.78	4.51
$0.98 < F/X \leq 1.020$	6.13	5.14	6.52	6.47	8.41	9.11
$1.02 < F/X \leq 1.050$	4.52	8.12	5.03	10.68	8.32	21.66
$1.05 < F/X$	3.17	15.09	3.79	18.13	7.53	45.75
Three-day estimation delay						
$F/X \leq 0.950$	19.43	4.27	20.55	4.46	41.42	4.98
$0.95 < F/X \leq 0.980$	9.06	4.54	10.69	5.17	11.32	5.79
$0.98 < F/X \leq 1.020$	7.01	7.16	8.03	8.06	10.43	11.82
$1.02 < F/X \leq 1.050$	4.69	11.18	5.28	13.32	9.41	26.21
$1.05 < F/X$	3.33	16.29	3.78	18.39	7.67	47.73
Four-day estimation delay						
$F/X \leq 0.950$	19.68	4.33	22.69	4.73	41.92	4.99
$0.95 < F/X \leq 0.980$	10.01	4.79	13.33	5.25	13.64	5.43
$0.98 < F/X \leq 1.020$	8.12	7.55	9.14	8.71	12.22	12.77
$1.02 < F/X \leq 1.050$	5.02	11.02	5.81	14.68	11.13	33.23
$1.05 < F/X$	3.64	18.42	3.83	21.15	7.79	51.31
All estimation delays						
$F/X \leq 0.950$	14.42	3.69	17.79	3.91	41.22	4.57
$0.95 < F/X \leq 0.980$	7.53	4.03	9.68	4.49	12.25	4.84
$0.98 < F/X \leq 1.020$	6.07	6.12	7.21	6.82	9.55	9.91
$1.02 < F/X \leq 1.050$	4.48	9.09	5.16	12.12	8.85	25.29
$1.05 < F/X$	3.25	15.44	3.89	18.26	6.77	47.67

D & H model stands for Doffou and Hilliard jump-diffusion stochastic interest rates model. Results are mean absolute percentage pricing errors for PHLX Deutschmark European futures options by moneyness, *F/X*, and estimation delay in days. Transaction periods are 7/1/84 to 8/31/89 and 3/1/95 to 12/30/99. All 40 contracts strike classes are included in the estimation and testing sample. Data are estimated on Mondays and the models are tested on Tuesdays, Wednesdays, Thursdays, and Fridays. This table shows the stability of the parameters estimates within the trading week.

Table F: Regression Analysis Of Pricing Errors For PHLX Deutschmark European Futures Options.

Regression parameters	D & H model Calls	Puts	Bates' model Calls	Puts	Black's model Calls	Puts
Intercept	- 57.92*	-3.67	- 52.83*	- 6.43	- 60.16*	- 113.38*
	(6.99)	(15.97)	(7.82)	(26.32)	(13.87)	(23.71)
F/X	89.63*	9.36	85.14*	9.11	49.79*	109.73*
	(11.08)	(13.01)	(11.49)	(21.57)	(20.42)	(27.69)
$(F/X)^2$	- 71.38*	- 11.91	- 52.97*	- 11.53	- 13.88	- 99.98*
	(6.67)	(8.77)	(10.47)	(11.21)	(10.64)	(19.21)
τ	2.69	7.89*	2.43	5.44*	- 0.09	0.97
	(1.67)	(2.58)	(1.74)	(2.19)	(2.25)	(2.95)
τ^2	- 9.75	- 6.22	- 7.33	- 16.79*	3.84	7.82
	(5.18)	(6.71)	(5.42)	(6.55)	(3.43)	(5.61)
R^2	0.030	0.033	0.034	0.060	0.49	0.17
F-test	19.18	14.92	23.76	17.11	669.73	101.19
Number of observations	15,644	10,025	15,644	10,025	15,644	10,025

The regression equation used is given below:

$$PE_i = \alpha_1 + \alpha_2(F/X)_i + \alpha_3(F/X)^2_i + \alpha_4\tau_i + \alpha_5(\tau_i)^2 + \epsilon_i, \qquad i = 1, 2,\ldots\ldots, n$$

where PE_i is the pricing error of each option, *F/X* is the moneyness, and τ is the time-to-maturity of the option. Standard errors (*not t-statistics*) are in parentheses and are calculated using the White heteroscedasticity consistent estimator. The regressions are run separately for calls and puts and cover all 40 contracts analyzed. Models are tested on Tuesdays with data from Mondays. Consequently, pricing errors are for Tuesdays, with parameters estimated from Monday data. The *t-statistic* is derived by dividing the regression coefficient by the standard error. Coefficients marked with an asterisk are statistically significant.