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TRANSACTIONS DATA EXAMINATION OF THE EFFECTIVENESS OF THE BLACK MODEL FOR PRICING OPTIONS ON NIKKEI INDEX FUTURES

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Abstract

Several recent studies have found that the Black (1976) model prices American options on futures quite accurately. These studies have used daily prices which are subject to non-synchronous trading. The present study uses transactions data on the Nikkei Index futures and options on Nikkei Index futures traded at Singapore International Monetary Exchange to examine the effectiveness of the Black model on an intra-day basis. The study finds that the model underprices both calls and puts. This is consistent with the fact that the model does not account for the early exercise feature of American options.

INTRODUCTION

Black's (1976) seminal article provides a framework for the analysis and valuation of commodity futures options. In his article he derives a model to price European options on forward contracts, which can be applied to European futures contracts, if the riskless rate of interest is constant during the life of the option. However, Black's model does not price the early exercise flexibility of American options. Baron-Adesi and Whaley (1987) compared hypothetical European and approximate American options on futures prices for six-month options. Their findings reveal that European out-of-the-money model prices were very similar to American futures option prices. However, they also found that in-the-money options reflect the benefit of the early exercise feature of the American options.

The modern dilemma is that while closed-form solutions do not exist for American options, (with the exception of cases where there is no value to early exercise), the options on futures currently traded on most world exchanges are American. During the last five to ten years, a large amount of theoretical work has concentrated on the problem of pricing these American futures options. Numerical methods, compound option methods, and quadratic approximations are a few of the methods presented in recent literature.

However, as Whaley (1986) points out, Black's model still has a wide range of applications. The model can not be applied exactly to most traded options, but practitioners continue to use the formula to estimate American futures options values and to calculate margin requirements, as it provides an easy-to-calculate, closed-form solution. Moreover, the more recent American futures options approximations are still subject to mispricing errors.

Whaley (1986) makes a comparison of traded prices and predicted model prices for options on S&P 500 futures. A summary of his results shows that overall, the deviations between actual market prices and theoretical model prices are not significant. There is however, some evidence that in-the-money options are underpriced by the model. More recent studies include those by Jordan, Seale, McCabe, and Kenyon (1987) on soybean futures options, and Bailey (1987) on gold futures option;, both of which find even smaller differences between market and model prices. Jordan and Seale (1986) and Blomeyer and Boyd (1988) examine Treasury bond futures options and find little evidence of differences between market prices and the prices predicted by the Black model. More importantly, most discrepancies were not large enough to be exploited. Ramaswamy (1985) finds that the value added by the American feature is rather small, especially for at-the-money options, despite the fact that premature exercise may be optimal.

A common question that emerges in recent literature is whether or not the discrepancies occur as a result of the market mispricing the options, or because the theoretical underpinnings of the model are incorrect. This analysis

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acknowledges that there are strengths and weaknesses in both the European Black model and the recent American approximations. The objective of this paper is to provide further empirical evidence on the performance of the Black model when applied to the dynamic financial environment of the modern era. This paper evaluates the performance of the Black model at predicting the prices of options on Nikkei 225 Stock Average Futures traded on the Singapore International Monetary Exchange (SIMEX) Limited. Most prior studies have examined the efficiency of the Black model using daily prices. These studies suffer from the problem of non-synchronous trading and as such, their results may not necessarily be valid. In this paper the efficiency of the Black model is examined using transactions data of Nikkei Index futures and options on futures. To eliminate the non-synchronous trading problem, only the sets of prices where calls, puts, and futures were traded within one minute of each other are considered.

SINGAPORE INTERNATIONAL MONETARY EXCHANGE

Established in September 1984, the Singapore International Monetary Exchange (SIMEX) is the first financial futures and options exchange in Asia. The international futures and options contracts listed by SIMEX represent a diversified product range covering interest rates, currencies, stock index, energy, and gold. SIMEX offers these trading opportunities to investors across the Asia Pacific and European regions, and (through the Mutual Offset System (MOS)) to interested parties in the US. All trading on SIMEX is executed on its trading floor via the open outcry auction system. In 1986 SIMEX became the world’s first futures exchange to launch the Nikkei 225 Stock Index futures. As a result of links set up with regard to common final settlement prices since then, SIMEX Nikkei 225 Average futures settle at the same price at the expiry date as the corresponding contracts at the Osaka Securities Exchange and the Chicago Mercantile Exchange (Lippo Group, 1994).

The Nikkei 225 Stock Average is a price-weighted series for 225 stocks on the First Section of the Tokyo Stock Exchange (TSE). The fact that the Index has shown stock price trends since the reopening of the TSE, makes it the most publicized and closely scrutinized share market series in Japan. A futures contract written on the Nikkei 225 Index is traded on the SIMEX floor. The Nikkei 225 Stock Average Futures contract value is 500 Yen times the Nikkei 225 Stock Average Futures Price, with a minimum price fluctuation of 5 index points (2,500 Yen), i.e. value of one tick is equal to 2,500 Yen. Contract months are: March, June, September, and December; listed on a quarterly cycle. The SIMEX futures option contract can be exercised to obtain one SIMEX Nikkei 225 Futures Contract. The contract months, expiration dates, and minimum price fluctuations are the same as the underlying futures contract. Strike prices are set at 500 Nikkei Index point intervals. At the beginning of trading of a new contract month, strike prices are listed above and below the previous days settlement price of the underlying futures contract. The options on futures are American, so that an option may be exercised on any day when the option contract is traded. Exercising an options contract will result in a futures position for the market participant.

DATA AND METHODOLOGY

Data

This investigation required the collection of transaction data for the Nikkei 225 Stock Average futures contract, transaction data for options on the Nikkei 225 Stock Average futures contract, and the Japanese “risk-free” interest rate data. The first two data categories were obtained from Simex intra-day transaction records from January 5, 1993 to June 30, 1993. For the futures contracts: the date, time, contract month, and traded prices: for the option contract: the date, time, contract month, option type, strike price, and traded prices were obtained. In both cases, only the nearest dated contracts in each quarter were chosen, as this is the most liquid contract. The risk-free rate was taken as the Japanese 90 day interest rate and this data were collected manually.

The next stage in the data collection involved the construction of a synchronous set of futures prices, and call and put option prices. This process involved closely matching in time, a set of option prices with the underlying futures contract. In sorting and matching the data, options and futures with the same maturity and traded within one minute of each other with the futures traded before the options were selected.

The final set of matched data then contained 1,160 call option and futures trades, and 767 put option and futures trades. Within the six month period, the number of matched trades on any given day ranged from 1 to 53. The average time between the futures and call option futures trades was 19 seconds, while the average was 24 seconds between futures and put option futures.

VOLATILITY ESTIMATION

The final variable needed as an input to the Black model is a measure of the standard deviation of the return on the futures contract. However, it is also the only variable which can not be directly observed, and is widely recognized as the most sensitive input parameter in pricing terms. A vast amount of literature has been produced on the topic of volatility estimation for pricing models. Most of these studies have shown that implied volatility is a more efficient ex-ante predictor of option prices than historical volatility.

A question that has arisen concerning the best measure of implied volatility is which of yesterday's option prices should be used to obtain a volatility estimate to value today's options? A range of approaches to utilizing implied volatilities have been presented in a number of empirical studies. Weighted average approaches have weighted each option's implied standard deviation according to: the partial derivative of each option's price with respect to its standard deviation (Latane and Rendleman, (1976)), the elasticity of the option price with respect to its standard deviation (Chiras and Manaster, (1978)), and each option's trading volume (Day and Lewis, (1988)).

Whaley (1982) has developed a regression method that aims to minimize the sum of the squared pricing errors to find the optimal or "best fit" estimate of implied volatility. Beckers (1981) suggests that the option that is closest-to-the-money is the one most sensitive to volatility changes and will give a measure of volatility as good as some of the suggested weighting schemes. In this study we use the closest-to-the-money volatility as the input to the pricing model.

The extent to which a call option is in the money is measured as:

Equation 1

$$M_{ji} = \frac{F_{ji} - K_{ji}}{K_{ji}}$$

The extent to which a put option is in the money is measured as:

Equation 2

$$M_{ji} = \frac{K_{ji} - F_{ji}}{K_{ji}}$$

where:

M = Moneyness
F_{ji} = Price of the futures contract
K_{ji} = Strike price

The volatility estimate obtained for day (i) is then used to value options on day (i+l). This implies that the first day's data for each contract will be used to calculate the volatility for the next day's trades, (thereby reducing the total data set for analysis). The primary focus of the analysis will then be to test the option pricing model described below.

THE PRICING MODEL

The model developed by Fisher Black (1976) can be used to price call options on futures;

Equation 3

$$C = e^{-rt}\,[FN(d_l) - KN(d_2)]$$

Equation 4

$$d_1 = \frac{\ln[F/K] + [0.5\sigma^2].t}{\sigma.t^{½}}$$

Equation 5

$$d_2 = d_1 + t^{½}$$

C denotes the price of a futures call option, F denotes the underlying futures price, K denotes the futures option exercise price, t is the time to expiry in years, r is the riskless rate of return, $N(.)$ is the standard normal distribution function, and σ^2 is the variance of returns on the futures contract.

In general, the pricing relationships explained for options on stocks also apply to options on futures; including the underlying assumptions concerning lognormally distributed prices, perfect and continuous markets, and interest rate certainty. However, a notable feature of this pricing model is that it does not include the risk-free rate as in the Black Scholes equation for valuing stock options does. This is simply a reflection of the fact that no immediate cash investment is required for the futures contract, as the margin can be posted in T-Bills, alternatively Black's formula can be viewed as the Black-Scholes with a continuous dividend yield, or cost of carry, equal to the risk-free rate.

The Black equation for put options on futures can also be formulated using put-call parity and equation (3) above so that:

Equation 6

$$P = Ke^{-rt} [1 - N(d_2)] – Fe^{-rt} [1 - N(d_l)]$$

where d_l and d_2 are defined as in equations (4) and (5) above.

STATISTICAL MEASURES

The sum and substance of the data analysis involves comparing the traded market prices to the theoretical premiums predicted by the Black model. This study adopts a common method of evaluating the performance of an option pricing model that involves calculating the following error metrics.

Mean Error

$$ME = \frac{1}{K}\sum_{j=1}^{K}(C'_j - C_j)$$

where C'_j is the predicted price of the option, and C_j is the actual price for observation j, and K is the number of observations.

The following measures are also commonly used to gauge the relative prediction error of the pricing model:

Percentage Mean Error (*PME*)

$$PME = \frac{1}{K}\sum_{j=1}^{K}[(C'_j - C_j)/C_j]$$

Root Mean Squared Error (*RMSE*)

$$RMSE = \sqrt{\frac{1}{K}\sum_{j=1}^{K}(C'_j - C_j)^2}$$

Median Absolute Percentage Error (*DAPE*)

$$DAPE = Median\ [(\ |C'_j – C_j|\ )\ /\ C_j]$$

Mean Absolute Percentage Error (*MAPE*)

$$MAPE = Mean\ [\ |C'_j – C_j|\ /\ C_j]$$

The data analysis also involves examining the option pricing model's pricing errors for systematic tendencies. In this analysis, the model is examined for moneyness bias and maturity bias.

To investigate moneyness bias the traded option prices are separated into three categories; *in-the-money*, *out-of-the-money* or *at-the-money*. In the case of maturity bias the data is also broken into three categories. *Short-term* includes options with less than 30 days to maturity, *medium-term is* between 30 and 60 days, while *long-term* includes options with more than 60 days to maturity.

RESULTS

Call Options

Table 1 presents the results of the error metric calculations for the entire sample. It is evident that the Black model underprices the futures options, with a mean error of -8.98. This underpricing provides evidence that the model ignores the early exercise facility, thus underestimating the value of the option.

TABLE 1
Error Metrics in Black's Model Using Nearest-the-Money Implied Volatility

Number of Observations	Mean Error ME	Percentage Mean Error PME	Root Mean Square Error RMSE	Median Absolute Percentage Error DAPE	Mean Absolute Percentage Error MAPE
1160	-8.98	-3.20%	42.36	4.96%	9.60%

A simple t-test can be carried out on the data set to test whether the model predicts prices correctly on average. The null hypothesis is simply that the mean pricing bias is zero. The results in Table 2 show that the t-statistic is much greater than the critical value, thereby causing the null hypothesis to be rejected. The mean bias is therefore significantly different from zero. This result does not provide further support for previous examples of efficient pricing by the Black model.

TABLE 2
Entire Sample Measures of Bias for Calls

Number of Observations	Mean Error	Standard Deviation	t-statistic	t-critical
1160	-8.98	41.42	7.38	1.65

It is recognized that implied volatilities will differ across the three moneyness groupings. It would therefore be expected that at-the-money option prices would be predicted most accurately given the nearest-the-money estimate of implied volatility used as an input to test the model. The results of the simple t-tests for each of the three categories confirms this expectation.

TABLE 3
Subsample Measures of Moneyness Bias for Calls

Sample	Number of Observations	Mean Error	Standard Deviation	t-statistic
Out-of-the- Money	564	-12.74	43.02	7.02
At-the-Money	425	-1.99	32.31	1.27
In-the-Money	172	-13.74	52.27	3.45

The smallest mean error for the predicted prices is -1.99, for at-the-money options. Moreover, the t-statistic is less than the critical value which confirms that the mean error is not significantly different from zero. Using the nearest-the-money volatility estimate to price the following day's options minimizes any strike price bias. Table 3 also shows the model significantly underprices both in-the-money and out-of-the-money futures options.

TABLE 4
Subsample Measures of Maturity Bias for Calls

Sample	Number of Observations	Mean Error	Standard Deviation	t-statistic
Short-Term	388	-4.12	25.17	3.22
Medium-Term	392	-2.34	39.87	1.16
Long-Term	383	-20.82	52.26	7.80

Table 4 above illustrates a fairly even spread of observations within each maturity category. The results also show consistent underpricing across all maturity categories. However, the t-test results (t = 1.16) show that the mean error is insignificant for medium term futures options. Long term futures options with greater than 60 days to maturity generate the greatest underpricing with a mean error of -20.82.

PUT OPTIONS

Table 5 presents the results of the error metric calculations for the entire sample. It is evident that the Black model once again underprices the futures options, with a mean error of -8.91. A comparison of Table 1 and Table 5 shows that the average underpricing by the model is similar for both put and call options. The difference in the mean errors is only 0.07, while there is a 11.23% difference in the percentage mean error. This underpricing provides further evidence that the model ignores the early exercise facility, thus underestimating the value of the option.

TABLE 5
Error Metrics for Black's Model Using Nearest-The-Money Implied Volatility

Number of Observations	Mean Error ME	Percentage Mean Error PME	Root Mean Square Error RMSE	Median Absolute Percentage Error DAPE	Mean Absolute Percentage Error MAPE
767	-8.91	-14.43%	28.05	8.88%	20.63%

The t-test (null hypothesis that the mean pricing bias is zero) results for the entire put option sample are presented in Table 6. The results show that once again the t-statistic is much greater than the critical value, resulting in the rejection of the null hypothesis. The mean bias is therefore significantly different from zero. The results for both put and call options on the Nikkei 225 Stock Average futures contract appear to be fairly consistent.

TABLE 6
Entire Sample Measures of Bias for Puts

Number of Observations	Mean Error	Standard Deviation	t-statistic	t-critical
767	-8.91	26.62	9.27	1.65

TABLE 7
Subsample Measures of Moneyness Bias for Puts

Sample	Number of Observations	Mean Error	Standard Deviation	t-statistic
Out-of-the-Money	480	-14.53	22.70	14.02
At-the-Money	252	-1.81	31.00	-0.93
In-the-Money	35	-9.04	17.66	3.03

The results of the moneyness category analysis once again meet expectations. At-the-money prices are again predicted most accurately by the model, reflecting the reasoning discussed earlier. A t-statistic of -0.93 means the null hypothesis cannot be rejected, so that the mean error is said to be insignificantly different from zero. However, for both in-the-money and out-of-the-money options, the model significantly underestimates the premium payment.

TABLE 8
Subsample Measures of Maturity Bias for Puts

Sample	Number of Observations	Mean Error	Standard Deviation	t-statistic
Short-Term	376	-8.50	21.27	7.75
Medium-Term	233	-9.82	23.36	6.42
Long-Term	162	-8.33	39.13	2.71

Table 8 above illustrates a declining number of observations from the short-term through to the long-term maturity category. However, the results once again show consistent underpricing across all categories. In each case the mean error is found to be significantly different from zero. Medium-term futures options have the largest mean error, which is in contrast to the insignificant pricing error found for medium-term call options.

REGRESSION ANALYSIS

This analysis involves regressing actual traded prices against the prices predicted by the Black model for both the call option and put option data sets. The analysis is based on the following regression equations;

Equation 7

$$C_i = \alpha_0 + \alpha_1 C'_I + \varepsilon$$

Equation 8

$$P_i = \alpha_0 + \alpha_1 P'_I + \varepsilon$$

A model with perfect prediction ability would produce regression coefficients of a equal to zero, and a_1 equal to one. The estimate of a_0 and the corresponding error provides a means of testing the degree of bias in the valuation model, while the estimate of a_1 and its standard error indicates the level of efficiency. The summary statistics for both data sets are presented below.

TABLE 9
Regression Coefficients

Sample	α_0	α_1	R^2
Calls	7.04	1.00	0.9974
Puts	16.34	0.97	0.9900

The results show the intercept is significantly different from zero (high degree of bias) for both types of options and the slope co-efficient is very close to the perfect value. For put options it appears the model tends to underprice low priced put options and overprice high priced put options (i.e. $a_1 < 1$). The model performs well for both data sets with explained variation exceeding 99% in both instances.

CONCLUSION

The aim of this research was to evaluate the performance of the Black model at predicting Option prices on Nikkei 225 Stock Average Futures traded on SIMEX on an intra-day basis. For call pptions using nearest-the-money implied volatility, the mean pricing error for the entire sample is negative and significantly different from zero. This is consistent with several previous empirical studies. The underpricing reflects the fact that the Nikkei 225 Stock Average American-style futures options traded on SIMEX have a higher value than the corresponding European options the model is based on. At-the-money options generated a (negative) mean pricing error not significantly different from zero as did medium-term options (30-60 days to expiry). Overall, both the maturity bias and moneyness bias was found to be monotonic with options in all data categories being underpriced. For put options using the nearest-the-money implied volatility, the mean pricing error for the entire sample is also negative and significantly different from zero. Again, the value of the early exercise facility of the American-style Nikkei 225 Stock Average futures option traded on SIMEX appears to be overlooked by the Black model. At-the-money options generated a negative mean pricing error not significantly different from zero. The model underpriced both in-the-money and out-of-the-money options. The maturity bias was again found to be monotonic as all three maturity categories were significantly underpriced.

When it is considered that the measure of implied volatility used as an input in the model was calculated using the option price closest-to-the-money, the moneyness bias conclusions are extremely logical. By using a volatility measure weighted towards at-the-money options, the model predicts the prices of at-the-money options most efficiently. Thus, this paper finds that even on an intra-day basis, the Black model underprices the options implying that the early exercise feature of American options is not being accounted.

The underpricing implications are that the Black model may not necessarily be an appropriate one to use for pricing American options. The market incorporates the value of the early exercise feature and reflects this in the prices of these options, examined on a transactions basis.

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