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A PEDAGOGICAL EXAMINATION OF THE RELATIONSHIP BETWEEN OPERATING AND FINANCIAL LEVERAGE AND SYSTEMATIC RISK

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INTRODUCTION

The study and understanding of risk is of paramount importance to any discussion of the value of a particular firm or enterprise. Most of basic financial management addresses risk from the perspective of a portfolio or the financial and operating characteristics of the firm. The two are often loosely connected and interrelationships ignored. This paper offers a pedagogical approach to the relationship between balance sheet decisions, product markets, and the systematic risk of a firm. This is accomplished by decomposition of the firm's beta. This decomposition extends earlier works and allows the reader to see the interrelationships mentioned above.

Financial theory is predicated on the notion that the goal of the firm is to maximize value and thus firms configure their balance sheets to achieve this goal. The selected configuration of assets and liabilities determines the total risk of the firm. Portfolio theory shows us that the relevant risk is the systematic risk as investors are able to diversify away the unsystematic portion of total risk. Several authors, in their study of systematic risk, have derived decompositions of beta providing insight into the financial and economic factors affecting beta. Hamada (1972) and Rubenstein (1973) have partitioned beta into operating risk and financial risk. Mandelker and Rhee (1984) provided an alternative decomposition.

The motive for the Mandelker and Rhee alternative decomposition was: (1) to explicitly introduce two types of leverage, operating and financial, (2) to avert various econometric problems (caused by a nonlinear, multiplicative effect of financial structure on risk as measured by the financially unlevered common stock beta) and (3) to avoid the assumption that corporate debt is risk free. The Mandelker and Rhee decomposition is built upon the assumption that the firm faces uncertainty as to the quantity demanded. This assumption is retained in order to include this market uncertainty in the extension developed in this paper. In this paper, the Mandelker and Rhee decomposition will be extended to assess the relationship of the firm's product demand elasticity with the firm's systematic risk and individual components. This extension is particularly relevant as it provides insight into how the firm's product market(s) affect the relevant risk measure.

The extension presented in this paper is consistent with the suggestion by Lindeberg and Ross (1981) that our understanding can be enhanced by studying the linkage between product markets and financial markets. Subrahmanyam and Thomadakis (1980), Hite (1977), Alberts and Hite (1983), Conine (1982, 1983) have also studied the linkage of the product and financial markets, however, in different settings and from a different point of view.

Introducing the demand elasticity and assessing the resultant implications leads to an alternative formulation of intrinsic risk to that delineated in Mandelker and Rhee. This alternative formulation brings into focus an expression that can be called a demand beta which appears to be an important source of uncertainty in a security's return.

BETA DECOMPOSITION

The equations presented in this section are abstracted from the complete derivation in the Appendix. Equation (1) shows the decomposition of beta into the elements: degree of operating leverage (*DOL*), degree of financial leverage (*DFL*) and its intrinsic business risk B^0.

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Equation 1

$$B_j = (DOL)\ (DFL)\ B^0$$

where $B^0 = Cov\ [(NI_{jt-1}/SR_{jt-1})(SR_{jt}/E_{jt-1}),Rmt] / Var(Rmt)$.

This equation introduces the relationship between the intrinsic business risk, net income (*NI*), sales revenue (*SR*) and the capital structure decision of the firm as represented by equity (*E*). Rewriting Equation (1) shows the business risk being primarily determined by the product of a firm's return on equity (*ROE*) and a sales revenue beta.

Equation 2

$$B^0 = [(ROE_{t-1})\ COV(SR_t / SR_{jt-1}, Rmt)] / Var(Rmt)$$

This fundamental relationship shows how the volatility of a firm's product market is a primary determinant of risk that is not easily avoidable.

RELATIONSHIP BETWEEN VARIOUS RISK DETERMINANTS

The initial beta decomposition,

Equation 3

$$B_j = (DOL)(DFL)B^0$$

shows the relationship between the equity beta, operating leverage, and financial leverage. Introducing a product demand function in Equation (4) provides the reformulation presented.

Equation 4

$$B_j = (ROE_{t-1})\ (1\text{-}u)\ Cov[Q_t/Q_{jt-1},Rmt] / Var(Rmt)$$

The importance of this arrangement is in the introduction of a product demand beta. It is crucial in examining the relationship between demand elasticity and leverage characteristics. In following the derivation in the Appendix, Equation (17A) is shown to express the relationship between *DOL* and product price elasticity.

Equation 17A

$$DOL / u = -(v / p) / (l - u)^2 < 0$$

where $u = 1 / [\ -(dQ / Q) / (dP / P)]$.

The partial derivative of this equation is negative with *u* and *e* inversely related. The partial relationship of *DOL* and the price elasticity of a firm's products is positive. This positive relationship shows theoretically how increased volatility of demand for a firm's product as determined by price will increase this portion of a firm's risk.

To fully develop the relationship between price elasticity and the firm's risk, we now look at the relationship with financial leverage. Differentiating Equation (3A) shows that no relationship exists between sales revenue and the degree of financial leverage. With the capital structure being a discretionary decision faced by management, this lack of a relationship is easily understood. However, the degree of the firm's financial leverage is an important determinant of the firm's overall risk.

To inspect the relationship between elasticity of demand in the product market and the systematic risk of the firm, equation is (5) is presented.

Equation 5

$$B_j = (DFL)\ (DOL)\ [ROE_{t-1}(1 - u)(Cov(Q_t / Q_{t-1}), R_{mt}) / Var(R_{mt})]$$

By examining the signs of the individual components of the right hand side of equation (5), we can determine the sign of the overall expression. For firms not experiencing distress, (i.e. *SR / NOI* and ROE_{t-1} are positive), we can focus on the covariance term. This covariance term divided by the $Var(R_{mt})$ can be considered as a demand beta. Hence equation (5) indicates the effects of the product market imperfection on the firm's level of systematic risk. The temporal instability of systematic risk is dependent upon the instability of demand in the product market.

DISCUSSION

The unique aspect of this presentation is its explicit introduction of the macroeconomic structure into the decomposition of the systematic risk beta. Specifically, the use of demand elasticity as an instrumental variable allows us to increase our knowledge of how the structure of the product market impacts systematic risk or beta.

The analysis demonstrates that a source of systematic risk for firm (i) is output market uncertainty. Equation (4) enables us to identify it. Equation (5) demonstrates that this uncertainty is magnified by (a) the asset structure, (b) the capital structure, and (c) the market structure, i.e. the degree of competitiveness within an industry as reflected by the demand elasticity of the firm's product. Thus, we can classify the factors effecting systematic risk into two groups:

a) A firm's decision variables, asset and capital structure, and
b) A firm's nondecision variables, the structure of the product market.

The simple identification of these factors increases our knowledge of how management decisions and exogenous economic conditions combine to affect risk and thus the process generating returns in the capital markets. Furthermore, it provides indications about the underlying causes of change in systematic risk over time. An example of these underlying causes would be innovative technology that results in changes of the demand structure. A practical application of this classification is that it can be of assistance in estimating betas of the common stock of the firm. For example, changes in the product market that signal changes in the demand for a firm's product also signal a change in the firm's systematic risk. Managers aware of these market changes and relationships can make decisions that will tend to have offsetting effects. Investors can assess these changes and actions to make decisions about portfolio adjustments.

The firm's non-decision variables are captured by the intrinsic business risk B^0 which represents the beta of sales revenue. B^0 is associated with the macroeconomic environment in which the firm exists and operates. It can be multiplicatively decomposed into the demand beta (B_{Demand}) and a function of the price elasticity of demand which in effect magnifies the uncertainty generated in the product market. Based on the analysis, the systematic risk of a common stock is the product of *DOL*, *DFL*, ROE_{t-1}, *1-u* and demand beta. Conceptionally, this decomposition is an extension of Hamada (1972), Rubinstein (1973), as well as Mandelker and Rhee (1984). It explicitly introduces the degrees of two types of leverage, and examines the effects of imperfections in the output market.

This study also derives a theoretical relationship between *DOL* and price elasticity. Conceptually, *DOL* represents the *K / L* (capital/labor) ratio employed by the firm. Demand elasticity magnifies the risk generated by the use of a given *K / L* ratio. Note that this relationship represents the impact of fixed cost on the systematic risk of the firm from the standpoint of the product market. However, the use of fixed cost considered from the input market affects the systematic risk of the firm in the opposite direction.

Therefore, a follow-up to this approach could study the impact of the fixed cost considered from the input market. This study demonstrates that the price elasticity of demand is a primary determinant of the absolute level of systematic risk and therefore partially determines a firm's cost of capital. The general expression in (5) examines the theoretical relationship between systematic risk and price elasticity. Making the reasonable assumption that the majority of firms demonstrate B_{Demand} positive, it follows that firms with higher (lower) monopoly power will exhibit lower (higher) betas. Therefore, it appears that monopoly power reduces beta. This conclusion leads to the empirical proposition, that high beta stocks are associated with competitive firms and low beta stocks with firms that have more market power in their output markets.

The decomposition presented in this paper is important as it provides insight on various relationships and helps in explaining the choices faced by management. The implications presented in this last section provide analysis of changes in beta and factors that should be considered by management as they change their balance sheet composition. Most of the implications of the model go beyond the typical textbook presentation, but should help students in understanding the interrelationships faced by management.

APPENDIX

Consider

Equation 1A

$$B_j = Cov\,(R_{jt},R_{mt}) / Var(R_{mt})$$

where:

R_{jt} = the rate of return on common stock *j* for the period from *t-1* to *t*.
R_{mt} = the rate of return on the market portfolio for the period from *t-1* to *t*.

By defining the returns to the *jth* security by $R_{jt} = (Ni_{jt} / E_{jt-1}) - 1$, if where NI_{jt} denotes earnings after interest and taxes at time *t* and E_{jt-1} represents the market value of common equity at *t-l*, and substituting this definition in formula (1A) while making use of the properties of the covariance operator, we have:

Equation 2A

$$B_j = (NI_{jt-1} / E_{jt-1})\; COV[(Ni_{jt} / NI_{jt-1}) - 1, R_{mt}] / Var(R_{mt})$$

The degrees of financial and operating leverage are defined as:

Equation 3A

$$DFL = [(Ni_{jt} / NI_{jt-1}) - 1] / [(NOI_{jt} / NOI_{jt-1}) - 1]$$

Equation 4A

$$DOL = [(NOI_{jt} / NOI_{jt-1}) - 1] / (SR_{jt} / SR_{jt-1}) - 1]$$

where:

NOI = Earnings before interest and taxes,
SR = Sales revenue
Q = the number of units produced and sold,

An alternative expression for *DOL* is:

Equation 5A

$$DOL = [(NOI_{jt} / NOI_{jt-1}) - 1] / [(Q_{jt} / Q_{jt-1}) - 1]$$

To use equation (5A), the following implicit assumption is made:

Equation 6A

$$(SR_{jt} / SR_{jt-1}) = (P_{jt}Q_{jt}) / (P_{jt-1},Q_{jt-1}) = (Q_{jt} / Q_{jt-1})$$

Conceptually (6A) can be considered true if firm *i* operates in a perfectly competitive product market. In this case $P_t = P_{t-1}$, for all *t*. Equations (4A) and (5A) will allow a decomposition, (in presence of a down-sloping demand curve), enabling the linkage of the product and financial markets.

Using equation (3A) we have:

Equation 7A

$$[(Ni_{jt} / NI_{jt-1}) - 1] = DFL(NOI_{jt} / NOI_{jt-1}) - 1$$

also from equation (4A) we have:

Equation 8A

$$[(NOI_{jt} / NOI_{jt-1}) - 1] = DOL(SR_{jt} / SR_{jt-1}) - 1$$

Substituting (8A) into (7A) yields:

Equation 9A

$$[(Ni_{jt} / NI_{jt-1}) - l] = (DFL)(DOL)[(SR_{jt} / SR_{jt-1}) - 1]$$

Substituting equation (9A) into equation (2A) yields:

$$Bj = (NI_{jt-1} / E_{jt-1}) Cov[(DFL)(DOL)[(SR_{jt} / SR_{jt-1}) - 1], R_{mt}] / Var(R_{mt})$$

Simplifying, we have:

Equation 10A

$$B_j = (DOL)(DFL) Cov[(NI_{jt-1})(SR_{jt}) / (E_{jt-1})(SR_{jt-1}), R_{mt}] / Var(R_{mt})$$

and

$$B_j = (DOL)(DFL)B^0$$

where:

Equation 10B

$$B^0 = Cov[(NI_{jt-1} / SR_{jt-1})(SR_{jt} / E_{jt-1}), R_{mt}] / Var(R_{mt})$$

Equation (10A) demonstrates the decomposition of systematic risk beta into its elements: (*DOL*), (*DFL*) and intrinsic business risk B^0. Rewriting (10a) and noting that:

$ROE_{t-1} = NI_{jt-1} / E_{jt-1}$, *the return on equity for the period t-1.*

We have:

Equation 11A

$$B^0 = (ROE_{t-1}) Cov[(SR_t / SR_{jt-1}), R_{mt})] Var(R_{mt})$$

Equation (11) demonstrates that the intrinsic business risk of a common stock B^0, is the product of the (*ROE*) for the last period with the beta of sales revenue $[Cov(SR_t / SR_{t-1}, R_{mt})]$.

We now introduce the product demand into equation (11A). Note that $1 / e = u = -[(P_t / P_{t-1}) - l)] / [Q_t / Q_{t-1} - 1)]$, thus $[(SR_t / SR_{t-1}) - 1] = [(Q_t / Q_{t-1}) - 1](1-u)$ and equation (11A) becomes:

Equation 11B

$$B = (ROE_{t-1})(1 - u)(Cov(Q_t / Q_{t-1}), R_{mt}) / Var(R_{mt})$$

The formulation of intrinsic risk represented by equation (11B) brings into focus the expression $Cov((Q_t / Q_{t-1}), R_{mt}) / Var(R_{mt})$ which can be thought of as a product demand beta. The importance of this beta will be discussed further after the relationship of the product demand elasticity with *DOL* and *DFL* is explored. Consider the continuous version of *DOL*:

Equation 12A

$$DOL = [d(NOI) / NOI] / [d(SR) / SR]$$

$$[(d(NOI) / NOI) / dQ / Q] / (1 - u), \text{ or}$$

$$DOL = Z / 1 - u$$

where:

$$Z = [d(NOI) / NOI] = [dQ / Q]$$

$$e = - [(dQ / Q)/(dP / P)] = \text{the price elasticity of demand}$$

$$u = - [(dP / P) / (dQ / Q)] = 1 / e,$$

Note that *NOI* is equal to:

Equation 13A

$$NOI = SR - vQ - F$$

where *SR* equals sales revenue, *v* equals per unit variable cost, and *F* equals fixed cost. The differential of equation (13A) is:

$$d(NOI) = d(SR) - d(vQ)$$

multiplying both sides by ($1 / NOI$) and rearranging the right-hand side yields:

$$d(NOI) / NOI = (SR / NOI)[d(SR) / SR-d(vQ) / SR]$$

Noting $PQ = SR$, we have:

Equation 14A

$$d(NOI) / NOI = (SR / NOI)[(dP / P + dQ / Q) - (vdQ / PQ)$$

From equation (14) it follows that:

Equation 15A

$$Z = (SR / NOI)(1 - u - v / p)$$

Equations (15A) and (12A) provide us with the following expression for *DOL*:

Equation 16A

$$DOL = [K(l - u - v / p)] / (1 - u)$$

where: $K = SR / NOI$

This expression relates *DOL* and price elasticity of demand. To examine the relationship between *DOL* and price elasticity we differentiate equation (16A) with respect to *u*. The partial derivative is:

Equation 17A

$$(DOL) / u = -(v / p)/(1 - u)^2 < 0$$

The theoretical relationship between demand elasticity and DFL is now considered. The differential of equation (3A) is:

Equation 18A

$$d(DFL) = NOI / NI, \text{ since } d(NI)=d(NOI)$$

Substituting equations (11A), (16A) and (18A) into equation (10A) and rearranging yields:

Equation 19A

$$B_j=[ROE_{j-1}(SR/NOI)_t(DFL)(1 - u - v / p)]Cov[(Q_t / Q_{t-1}), R_{mt}] / Var(R_{mt})$$

Alternatively, equation (19A) can be written as:

$$B_j= (DFL)(DOL)[ROE_{t-1}(1 / u)[Cov(Q,R_m) / Var(R_m)]$$

To examine the relationship between systematic risk and price elasticity we differentiate equation (19A) with respect to *u* as follows:

Equation 2OA

$$B_j / u = -(SR / NI)ROE_{t-1}\ Cov[(Q_t / Q_{t-1}, R_{mt}) / Var(R_{mt})$$

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