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INSURANCE CONTRACT VALUATION, EXPERIENCE RATING, AND ASYMMETRIC INFORMATION

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Abstract

The study follows previous work on the role of experience rating in resolving information asymmetry problems in insurance contracting. Results of the study provide additional evidence that contracts which are experience-rated are less susceptible to pricing inefficiency due to moral hazard and adverse selection than are contracts which are not experience-rated. This study extends previous work by showing that experience-rated contracts are most effective among lower risk insureds.

INTRODUCTION

In insurance contracting the insurer assesses *ex ante* the expected loss of the individual insured. Ideally, the insurer observes the true risk of loss for each individual, losses are due to states of nature beyond the control of the insured, and the insurance contract price reflects the expected loss costs of the insured. However, in practice the insured's true risk level is not directly observable by the insurer, and problems of adverse selection and moral hazard arise. Adverse selection occurs when the insured has more information *ex ante* than the insurer about the insured's true risk level. Moral hazard exists when the insured changes behavior in a way which increases the probability or severity of loss after the contract has been written so that the contract price no longer reflects true expected loss costs. That is, adverse selection arises from hidden knowledge and moral hazard arises from hidden action (Riley, 1985).

This study follows previous work on the role of experience rating in resolving information asymmetry problems in insurance contracting.[1] Results of the study provide additional evidence that contracts which are experience-rated are less susceptible to pricing inefficiency due to moral hazard and adverse selection than are contracts which are not experience-rated. This study extends previous work by showing that experience-rated contracts are most effective among lower risk insured's.

In Section II a simple, single-period insurance contract is developed using option pricing methodology in a *perfect information* environment in order to establish fair prices based on the insured's true expected loss. The contract then is augmented whereby the insurer pays a cash amount to the insured at the end of the contract period in the event that no claim is made, and the value of the augmented contract then is derived. Using an *ex post* discount captures the essence of experience rating where subsequent contract prices are adjusted to reflect observed loss experience.

Both the simple contract and the augmented contract (i.e., the experience-rated contract) are then analyzed given *imperfect information* to allow the effects of moral hazard and adverse selection to be investigated.[2] The augmented contract is shown to be less subject to pricing inefficiency due to information asymmetries. In Section III the role of the discount feature is more extensively investigated through the simulation of contract values over a wide range of expected losses. Experience rating is shown to be most effective in controlling pricing inefficiency due to moral hazard and adverse selection among lower risk insureds. Section IV summarizes the results of the study.

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INSURANCE CONTRACT PAYOFFS AND PUT OPTIONS

In this section the analogy between insurance contracts and put options in the capital markets is described in a *perfect information* setting.[3] Sufficient assumptions are invoked on loss distributions and the insureds' utility functions to allow insurance contracts to be valued by the option pricing model developed by Black and Scholes (1973). As with any theory, some generality is lost in appealing to restrictive assumptions such as those needed to derive the Black-Scholes model, but the advantage gained is that insights previously obtained only from multiperiod models of insurance contracting become transparent in this simple framework. The Black-Scholes model is then extended to allow for *ex post* discounts. Finally, both contracts are examined in an *imperfect information* setting to allow investigation of the effects of moral hazard and adverse selection across a broad range of risk classes.

Insurance As A Put Option Given Perfect Information

A put option is a financial asset that entitles the buyer of the option to force the seller to purchase an underlying asset at an agreed upon price (the exercise or striking price). Any form of insurance coverage may be viewed as a put option. Consider, for example, an insurance contract for an automobile with an insured value of $15,000. If, during the contract period, the automobile is damaged and its value drops to $11,000, the insured may exercise the put option. The insurer will be required to pay the insured the difference between the insured value ($15,000) and the actual asset value ($11,000), or $4,000 (less the deductible). The $4,000 loss is borne by the insurer. Had there been no damage, the automobile would have been worth at least $15,000 (ignoring depreciation or appreciation), and the insured would not have exercised the put option.

A well-known pricing formula for put options in the capital markets was developed in the early 1970s by Black and Scholes (1973). In order to apply the Black-Scholes formula to value insurance contracts, the following assumptions are made. The contract is written for a single period of length t, where T denotes the ending date of the period. This assumption appears reasonably consistent with actual insurance practice. The second assumption is that the contract may be exercised only on date T and not before. This restriction clearly deviates from observed practice.[4] The assumption is needed in order to use the Black-Scholes model in an insurance setting but does not invalidate the essential results that follow.[5,6]

The insured asset's value (VT) at the end of the contract period is assumed to be a log-normal random variable, where the variance of asset value as of date T is denoted as $\sigma^2 T$.[7] The distributional restriction is important since without it the Black-Scholes model does not hold. Other pricing models may be derived under different distributional assumptions,[8] and future research will determine if another choice would alter the conclusions.[9]

The asset is insured for the amount I and both the insured and insurer are *risk neutral*.[10] This is clearly an objectionable assumption in an insurance setting since risk aversion is essential for rational purchase of insurance. However, the assumption in this case does not alter the essential results as will be demonstrated below.

The above conditions are sufficient for the insurance payoffs to be valued according to the Black-Scholes European put option formula as follows (see Cox and Ross (1976)):

Equation 1

$$P = I\, e^{-rt} N(-d_2) - VN(-d_1)$$

In equation (1), V denotes the current value of the asset and N(.) denotes the cumulative standard normal density evaluated at $-d_2$ or $-d_1$. These are in turn defined as:

Equation 2

$$d_1 = \frac{\ln (V/I) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

Equation 3

$$d_2 = \frac{\ln (V/I) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}$$

The symbol r denotes the constant risk-free rate of interest that is assumed to hold at every instant during the contract period. The risk-free rate enters the model equation (1) due to the risk-neutrality assumption. Note, however, that a *risk averse* insured would simply use a higher risk-adjusted discount rate.

The option pricing framework, like traditional actuarial pricing models, develops a premium based on expected loss. Given risk neutrality, an agent will pay an amount for an insurance contract equal to the present value of expected loss. Realized loss is I-V, thus contract value is the present value of the expected loss conditional on a loss having occurred. The conditional expected loss is expressed as:

Equation 4

$$E[I - V \mid V < I] \times \text{Probability of Loss}$$

In equation (4), the probability of loss assuming V is log-normal is the cumulative standard normal density evaluated at $-d_2$, which is $N(-d_2)$ in equation (1).[11] This is also the probability that a put option will be exercised (see Jarrow and Rudd (1983)). When the expectation $E[I - V \mid V < I]$ in equation (4) is evaluated under the log-normality assumption for V, the loss probability is expressed as $N(-d_2)$, and the resulting expression is multiplied by the present value factor e^{-rt}, the result is equation (1). Thus, equation (1) is the present value of expected loss conditional on a loss having occurred, when the insured asset value is log-normally distributed and agents are risk-neutral.

Example: Let I = \$15,000, V = \$15,000, t = 6 months = 1/2 year = .5, r = 8 percent = .08, and $\sigma^2 = .025$. Carrying out the calculations implied in equations (2) and (3), we find $d_1 = .4140$ and $d_2 = .3021$. Next, evaluate the cumulative standard normal density N(.) at $-d_1 = -.4140$, then at $-d_2 = -.3021$. The resulting values are N (-.4140) = .3394 and N(-.3021) = .3813, approximately. Then equation (1) becomes:

$$P = 15000\, e^{-.08 \times .5} (.3813) - 15000 (.3394) = \$404.24$$

In summary, equation (1) is the actuarially fair price that a risk-neutral insured would be willing to pay for a contract given the parameters V, I, σ^2, r and T. It is important to note that given the parameters V, I, r, and T, the insured's expected loss is a positive, monotonic function of the variance parameter σ^2. Thus, we can speak of changes in expected loss interchangeably with changes in σ^2. Further discussion of the link between σ^2 and expected loss appears below.

Augmented Put Options Given Perfect Information

The model is now extended to allow for experience rating where the insured receives a discount, paid retroactively, if no claim is made during the coverage period. This differs from the way in which experience rating is administered in practice, where price adjustment is made on subsequent contracts. Nonetheless this *ex post* discount mechanism captures the essence of the feature without the complications of a multiperiod setting. Having the insured receive a cash payment at date T if no claim has been made is the same as offering a contract at a reduced price for the following period, and there is no loss in the generality of the results from using a single-period model.

Assume that the discount is a fixed amount paid according to the following schedule:

Equation 5

$$Discount = \begin{cases} \theta I & if\ \infty \geq V_T > I(1-\theta) \\ 0 & V_T < I(1-\theta) \end{cases}$$

In equation (5) the discount is a fixed proportion (θ) of the coverage amount (I). For example, for an automobile insured for $10,000, θ might be .01, thus the *ex post* discount would be .01 x 10,000 = $100. According to equation (5) the discount will be forfeited if V_T is below I(1-θ). For instance, if the automobile insured for $10,000 is in a collision and its value (V_T) declines to $7,000, the insured will file a claim for I - V = 10,000 - 7000 = $3000 and forego the discount. Suppose, on the other hand, that the loss was minor, say $50. Thus V_T = $9950 and the insured could choose to submit a claim for I - V = $50, but this would mean forfeiting the discount of $100. Thus, the insured will file a claim only if V_T is less than I(1-θ) = 10,000(.99) = $9900.

The payoffs to the insured with the augmented contract is given by equation (6).

Equation 6

$$Payoff = \begin{cases} I - V & if\ V_T < I(1-\theta) \\ \theta I & V_T \geq I(1-\theta) \end{cases}$$

The value of this contract (P*) is given by equation (7).

Equation 7

$$P^* = Ie^{-rt} N(-h_2) - VN(-h_1) + \theta Ie^{-rt} N(h_2)$$

where:

Equation 8

$$h_1 = \frac{\ln (V/I(1-\theta)) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

and

Equation 9

$$h_2 = \frac{\ln (V/I(1-\theta)) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}$$

The contract with the *ex post* discount is an augmented version of the put option formula in equation (1). The first two terms on the right-hand side of equation (7) represent the Black-Scholes put option formula with exercise price I(1-θ). That is, the insured will claim payment for a loss as long as the loss exceeds V - I(1-θ). The third term on the right-hand side of equation (7) can be interpreted as the present value of the expected discount payment. The probability that V_T > I(1-θ) is the probability that a claim will not be filed, and under the restriction in footnote 12 this probability is N(h_2). Thus, the straight contract in equation (1) and the augmented contract in equation (7) differ in two respects. First, the boundary condition for rational exercise of the put options differs (i.e., V_T < I for the straight contract and V_T < I(1-θ) for the augmented version). Second, the augmented contract in equation (7) contains the present value of the expected discount payment. Note that given risk neutrality and perfect information, the insured would be indifferent between the two contracts priced at P and P* since both are actuarially fair.

Insurance Contracts Given Information Asymmetries

In this section we consider the effects on contract values of incorrect assessment of the variance parameter σ^2. Recall that the variance parameter σ^2 in the model is tied directly to expected loss, and increases in σ^2 cause increases in expected loss. Thus, although traditional analysis of adverse selection and moral hazard deals directly with the insured's expected loss, here the focus is on σ^2 to allow a more straightforward interpretation of the effects of information asymmetries on expected loss.

Suppose that the perfect information assumption is relaxed and the insured knows the true variance or risk of the asset but the insurer does not; that is, potential for adverse selection exists. If the insurer underestimates variance of the asset, the contract will be underpriced as may be seen by taking the derivative of P in equation (1) with respect to σ^2:

Equation 10

$$\frac{\partial P}{\partial \sigma^2} = Ie^{-rt}\, Z(d_2)\, \sqrt{T}\ /\ 2\sigma > 0$$

In equation (10), Z(.) is the standard normal density evaluated at d_2 and it is clear that equation (10) is unambiguously positive. If adverse selection exists so that the assessment of σ^2 is too low, then the insured would be able to purchase coverage for less than the actuarially fair value. Similarly, if moral hazard exists so that σ^2 can be altered by the insured after the contract has been written, then P is not an actuarially fair price for the unaugmented contract. It is in this sense that adverse selection and moral hazard are modeled in the study. Given information asymmetries, the unaugmented contract leaves the insurer vulnerable to prices which are below expected loss costs; that is, pricing inefficiency results from information asymmetries.

A change in σ^2 affects the value of the unaugmented contract (P) differently than it does the value of the contract (P*) in equation (7) which is augmented with the experience-rated discount provision. This is illustrated by differentiating P* with respect to σ^2 as follows:

Equation 11

$$\frac{\partial P^*}{\partial \sigma^2} = I(1-\theta)e^{-rt}\, Z(h_2)\sqrt{T}\ /\ 2\sigma + e^{-rt}\theta\, I\left(\frac{1}{\sqrt{2\pi}}e^{-h^2}\ /\ 2\right)\frac{\partial h_2}{\partial \sigma^2}$$

The comparative static result in equation (11) is of lower value than the result for the straight contract in equation (10). This is seen by first comparing equation (10) with the first term on the right-hand side of equation (11). It is clear that $I(1-\theta)$ in equation (11) is less than I in equation (10) for $\theta > 0$. And $Z(h_2)$ in equation (11) is less than $Z(d_2)$ in equation (10) because h_2 (equation (9)) is greater than d_2 (equation (3)) for $\theta > 0$. Thus, the first term on the right-hand side of equation (11) is generally less than equation (10), and the remaining term in equation (11) is *negative* ($\partial h_2/\partial\sigma^2 < 0$ by inspection of equation (9)).

Thus an increase in value of the augmented contract due to an increase in expected loss (σ^2 in the model) is generally lower than the increase in an otherwise identical contract price (which is not experience-rated) induced by the same change in σ^2. This is illustrated in the following example.

Suppose an automobile is insured for \$5000 (I) for 6 months (t = ½ year = .5), and the car has a value at the beginning of the period of \$5000 (V). The interest rate is 8 percent (r = .08) and the risk of the car is assessed by the insurer such that σ is .45. The discount paid in the event there is no loss is \$65 (θ = .013). Using these values in equation (1) and equation (8) gives the respective values of the unaugmented contract (P) and the augmented experience rated contract (P*):

P = \$526.42
P* = \$557.39

In this risk-neutral setting an insured with $\sigma = .45$ would be indifferent between the two contracts. Each has an actuarially fair price: P = \$526.42 and P* = \$557.39. Suppose that the true expected loss of the insured was greater than that assessed by the insurer. (Recall that in this model higher expected loss can be generated by an increase in σ.) Let the true value of the parameter, σ, be .55 instead of .45. Carrying out the calculations in equation (1) and equation (7) yields the following:

P = \$661.65
P* = \$691.02

By increasing risk (σ) to reflect the insured's true level of expected loss in the example, the contract value without the discount feature increases by approximately 26% ((661.65 -526.42)/526.42). That is, an insured who contracts for $\sigma = .45$ pays only \$526.42 for a policy that should be priced at \$661.65 given the true expected loss. For the augmented policy the percentage increase is only about 24% ((691.02 - 557.39)/557.39). Thus, the gain to the insured from concealing the true higher expected loss is less per dollar of contract value when the contract has a discount feature.

An increase in σ^2 unambiguously increases the value of the unaugmented version of the contract to the insured, as was shown in the comparative static result in equation (10). This is because higher dispersion in the probability distribution of V_T increases expected loss. Since the payoff for $V > I$ (no loss) is zero without the discount, the insured receives a contract for P that is worth more than this amount if expected loss is higher than that assessed by the insurer. With the discount feature, however, increasing expected loss has two effects on the insured. As σ^2 increases, the expected loss increases as before, but the probability of receiving the discount declines. The two effects offset each other, so as to render ambiguous the sign of $\partial P^*/\partial\sigma^2$ in equation (11), and it is always true that $\partial P/\partial\sigma^2$ is greater in absolute value than $\partial P^*/\partial\sigma^2$.

In summary, given a world with perfect information, insurers and insureds would be indifferent between the contract without experience rating (priced at P) and the contract with experience rating (priced at P*) since both are actuarially fair. However, in a world with information asymmetries, the contract without experience rating leaves the insurer more vulnerable to inaccurate pricing, and the experience rated contract clearly dominates. The model is thus consistent with other theoretical developments (for example, Rubinstein and Yaari (1983)). The main result of the study, that experience rating is more effective in reducing pricing inefficiency among lower risk insureds (i.e., insureds with lower expected losses) is shown in the next section of the paper.

SIMULATION ANALYSIS

In this section the role of the discount feature is more extensively investigated through simulation of contract values over a wide range of σ values. This will allow comparison of the effects of experience rating in reducing pricing inefficiency across classes of insureds with different expected losses. Table 1 reports contract prices (P) without the discount feature and prices (P*) for contracts with the discount, for values of σ ranging (in .05 increments) from .05 to .95. The values P and P* are from equations (1) and (7), respectively, and are based on the following parameter values: $V = 10{,}000$, $I = 10{,}000$, $r = .08$, $t = .5$, $\theta = .01$. These parameter values are chosen to represent a typical automobile insurance contract. Note that the price with the discount, P*, is simply the present value of the expected experience rating discount of \$100.00 plus the actuarially fair premium for the insured's level of risk.

The table demonstrates that the contract would be underpriced if the insured's true risk level (hidden from the insurer) was actually ($\sigma + .05$) or greater. The column, Percentage Change From Preceding Value, shows a relative measure of the pricing inefficiency. An insurer, however, could offer P*, the contract with the *ex post* discount, so that incentives would exist for the insured to maintain (not hide or alter) σ, thus decreasing the potential mispricing in contract value from adverse selection or moral hazard.

TABLE 1
Values Of Insurance Contracts With And Without A Discount Provision

	Contract Without Discount		Contract With Discount	
σ[1]	Price Without Discount (P)	% Change From Preceding Value Of P	Price With Discount (P*)	% Change From Preceding Value Of P*
.05	22		108	
.10	124	463.64%	193	78.70
.15	247	99.19	310	60.62
.20	379	53.44	436	40.65
.25	512	35.09	566	29.82
.30	647	26.37	699	23.50
.35	782	20.87	932	19.03
.40	917	17.26	966	16.11
.45	1,052	14.72	1,100	13.87
.50	1,188	12.93	1,234	12.18
.55	1,323	11.36	1,368	10.86
.60	1,458	10.20	1,502	9.80
.65	1,592	9.19	1,635	8.85
.70	1,726	8.42	1,768	8.13
.75	1,860	7.76	1,901	7.52
.80	1,993	7.15	2,033	6.94
.85	2,125	6.62	2,164	6.94
.90	2,256	6.16	2,295	6.05
.95	2,387	5.81	2,425	5.66

Parameter values are: $V = 10{,}000$, $I = 10{,}000$, $r = .08$, $t = .5$, $\theta = .01$.
1. σ represents standard deviation of rate of change in asset value.

Results from Table 1 demonstrate that the percentage increase in contract value for the policy without the discount provision always exceeds the percentage increase with the discount. For example, where σ increases from .25 to .30, the value of the contract without the discount increases 26.37 per cent, whereas the value of the contract with the discount increases only 23.54 percent.

However for higher risk insureds (σ = .80 to .95 in the example), the percentage increases in contract value are smaller than for lower risk insureds (as seen by comparing values in the third and fifth columns). That is, the experience rating feature has its greatest effect in reducing mispricing among lower risk insureds (σ = .10 to .75). For low-to-moderate risks, experience rating has a greater effect because these insureds face a relatively high probability of receiving the discount. The discount is not a negligible component of the total price of the contract (premium plus discount, if any), and a change in the probability of receiving the discount significantly affects contract value. High-risk insureds face a relatively small probability of receiving the discount, which makes the contract with a discount much like the unaugmented contract.[12]

There is anecdotal evidence supporting the theoretical result that experience rating is more effective in reducing pricing inefficiency among lower risk insureds.[13] However, future empirical research remains to be done to test this in the automobile insurance marketplace.

SUMMARY

Consistent with previous theoretical and empirical studies, the model shows that experience rating can limit contract pricing inefficiency due to asymmetric information. The study further shows that experience rating is most effective in reducing pricing inefficiency among lower risk insureds.

ENDNOTES

1. See, for example, Rubinstein and Yaari, 1983, and Dionne and Lasserre, 1987.

2. Following Laffont and Tirole (1986) and Dionne and Lasserre (1987), adverse selection and moral hazard are considered simultaneously.

3. See Smith (1979) for details.

4. This restriction makes the insurance contract analogous to an European put as opposed to an American put which may be exercised at any time during the coverage period.

5. The assumption ruling out early exercise can be dropped under fairly general conditions in the case of *call* options traded in the capital markets, as shown formally by Martin (1973). This is not true for put options and the only methods known for valuing put options with early exercise features require cumbersome numerical solutions.

6. This assumption also rules out filing multiple claims during the contract period. Multiple claims could be accommodated in the model by requiring that they be combined and filed only on date T, as long as losses suffered during the period before T do not alter the distribution of losses on date T.

7. Formally stated, σ^2 is the instantaneous variance of the rate of growth in asset value over the contract period. Assuming that changes in asset value during the period are independent, the variance as of the end of the period is:

$$\int_O^T \sigma^2 dt = \sigma^2 T$$

8. A similar model can be derived using virtually any distribution for which the expected value conditional on a loss exists.

9. The authors' conjecture at this point is that the classes of distributions under which the results continue to hold are numerous.

10. While risk neutrality is not needed to derive the Black-Scholes model, use of alternative assumptions would also be restrictive. Black and Scholes (1973) and Merton (1973) derive the model assuming risk aversion, but investors are assumed to be able to form special portfolios in the capital markets that hedge against risk. Rubenstein (1976) relaxes the requirement for perfect hedging and shows that the Black-Scholes model continues to hold given risk averse investors.

11. Strictly speaking, this statement implies that the instantaneous expected rate of change in V is equal to r, the risk-free rate.

12. For example, where σ = .95, the unaugmented contract is worth \$2,387 and the augmented contract is worth \$2,425.

13. Based on discussions with Mr. J. Smith Harrison, Jr. of Seibels Bruce Group, Inc.

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